Sparse Coding

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March 16, 2016
Sparse coding is a method of representing images as signals composed of a few basis functions out of an overcomplete set added together. Similar to Fourier Transforms, but sparse. Two main types of support for sparse coding hypothesis:

- Biological support
- Statistical support

Sparse coding is important to understand the brain as well as to process images efficiently with little information.
The imaging model used for Sparse Coding is:

\[ I(\vec{x}_j) = \sum_{i=1}^{n} a_i \phi_i(\vec{x}_j) + n(\vec{x}_j) \]

Where \( I(\vec{x}_j) \) is an image patch at position \( x_j \), \( \phi_i \) is a basis function, \( n \) is noise and \( a_i \) is amplitude.
Many different kinds of superresolution
Looking at deblurring and denoising images specifically
Extremely useful for medical imaging, fluorescent microscopy, fingerprint scanning, etc. where you want to take a low quality image and make it better.
What do I mean when I say noise?

- Noise is a grainy disturbance in an image’s signal.
- Each pixel in an image is represented by a number and noise is an addition or subtraction to each of those pixel’s numbers.

If we consider each pixel to be some value $a_i$ then the corresponding pixel on the noisy image would be $a_i + \nu$. There are two main types of noise that the research is concerned with:

- Additive White Gaussian Noise (AWGN)
- Poisson Noise

Noise exists whenever images are involved.
The bulk of the research on Sparse Coding is done with AWGN. Referring back to the $a_i + \nu$ model, $\nu$ in this case would have a probability distribution of:
Poisson Noise is similar to Gaussian noise. Poisson noise is much more applicable in some situations (low light). Poisson isn’t symmetric like Gaussian. Its distribution looks like:

![Poisson Probability Distribution](image)
Examples of Noise

Original Photo Credit: Dorothea Lange, 1936
Examples of Noise

Sparse Coding
Examples of Noise

Sparse Coding
Blur Overview

Blur generally occurs because of movement or focusing issues.

Original Image

Motion Blur at 45°

Average Blur
Deblurring motion blur using least squared norms:
Why are we looking at superresolution?

- The regularization in sparse coding works, but could use some improvement as imaging problems are ill posed.
- Super resolution research also has to deal with image problems being ill posed, some methods work really well.
- We want to deal with more noise in images
- Super resolution research is helping us understand other ways to deal with the image problems being ill posed.
Deblurring algorithm shown earlier uses something called a least-squared norm.

Has been shown to be very effective in denoising as well when paired with wavelet frames.

Wavelet frames induce a sparse representation of an image using specific basis functions (e.g. Haar, Daubechies, Coifman).

Least Squared norms have two norms and have shown to be very effective in super resolution.
I’d like to thank:

- Dr. Luo
- Linfield Student Faculty Collaborative Research Grant
- Mathworks for the algorithms that I’ve been learning from