Let $G = (V, E)$ be a simple graph with no isolated vertices, and let $n \in \mathbb{N}$. For each $v \in V$, let $N(v)$ be the set of edges at $v$. We define a labeling of $G$ using elements of $\mathbb{Z}_n$, called a modular edge-sum labeling, in the following way:

- Assign a label from $\mathbb{Z}_n$ to each edge $e \in E$, denoted $\omega(e)$.
- For all $v \in V$, define the label of $v$ in the following manner:
  $$\ell(v) = \sum_{e \in N(v)} \omega(e),$$
  where the summation is computed modulo $n$.

A 1-relaxed edge-sum labeling of a graph $G$ is for each labeled vertex $v \in V$, $v$ has at most one neighbor vertex that has the same label. For any turn $\ell$ of the game, we define the label for $G$ where this sum is computed modulo $n$.

Let $L = (V, E)$ be the set of labeled edges. For any turn $\ell$, we have $\Lambda(v) = \sum_{e \in N(v)} \omega(e)$, and we require that the players maintain a legal 1-relaxed edge-sum labeling at each stage of the game.

Let $G$ be a simple graph with no isolated vertices, and let $L \in \mathbb{Z}_n$ be a labeled vertex for which $\omega(e)$ is the label of $e$. We define a 1-relaxed edge-sum labeling in $G$.


References