Tilings of Annular Regions

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Main Result

Theorem 1. \( A_2(a,b) \) is tileable by \( T \) if and only if
- \( n \) is even,
- \( n = 3 \) and \( a = b \) (mod 2), and it is not the case that \( a, b = 0 \) (mod 4),
- \( n \geq 5 \) is odd and \( a = b \) (mod 2).

The theorem above completely classifies which of our \( A_2(a,b) \) regions are tileable by \( T \). Below we present a proof of the first case when \( n \) is even. To do this we first develop a few definitions and lemmas.

Definition 1. An extended \( T, X_n \), is any rotation of a region formed by removing the two corner squares from first row of a \( 2 \times n \) rectangle for all \( n \geq 3 \) (see Fig. 3).

Lemma 2. The region \( X_n \) is tileable by \( T \) for all odd \( n \geq 3 \).

Proof. We proceed by induction. For \( n = 3 \) we have a \( T \) tile. Now assume \( X_n \) is tileable by \( T \) for some odd \( n \geq 3 \). Now we must show that \( X_{n+2} \) is tileable by \( T \). Simply add a skew tetrominoe to one end of the region. The remaining area is tileable by the induction hypothesis.

Figure 3: An extended \( T \) of length \( n \)

Figure 4: Illustration of the induction step.

Figure 5: An arrangement of odd length extended \( T \)'s showing that the region is tileable \((a,b = 0 \mod 2)\).

From these lemmas, it is not difficult to show that \( A_2(a,b) \) is tileable by \( T \) for all even values of \( n \). This can be done by induction on \( n \). The previous lemma provides the base case.

Other Results

- We proved that the tile counting group for the \( A_2(a,b) \) regions with respect to \( T \) is isomorphic to \( \mathbb{Z} \times \mathbb{Z} \). This means that we know what all the tile invariants look like for these regions with our tile set.
- We were able to show that the extended \( T \)’s have a local move property.
- We also proved that there are \( 2^{n-1} \) ways to tile \( A_2(a,b) \) by \( T \).

References

[4] C. Lester, Tiling with \( T \) and \( T \)-skew \( T \)-trominos, Querquet: Linfield Journal of Undergraduate Research, Vol. 1: Iss. 1, Article 3