Optimal Race Strategies for Cyclists

Math Modeling

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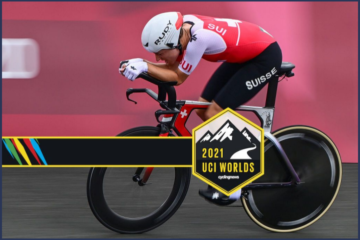


OUTLINE

Introduction

INTRODUCTION

Objective:
Model the optimal race strategy for a given cyclist and apply it to several courses.
Considerations:

- ◇ **Factors:** change in elevation, curvature of course
- ◇ **Courses:** 2021 Tokyo Olympics Individual Time Trials, 2021 UCI World Championships Individual Time Trials, and Le Tour de Mac
- ◇ **Methods:** Use of The Skiba Energy Store Model and Applied Kinematics

The Skiba Energy Store Model

THE SKIBA ENERGY STORE MODEL

Equations

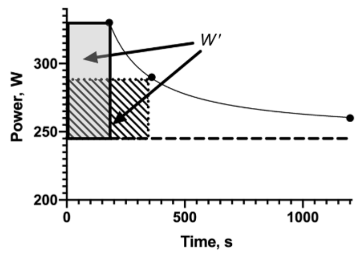
$$P = \frac{W'}{t} + CP. \tag{1}$$

$$W'_{\text{bal}} = W' - \int_0^t W'_{\text{exp}} e^{\frac{-(t-u)}{\tau_{w'}}} du \tag{2}$$

- ◇ P: instantaneous power output
- ◇ CP: the upper limit of an athletes power output sustainable for a given interval of time
- ◇ W': the amount of work that can be done above a rider's CP for a given amount of time
- ◇ W'_{bal}: ”balance” between the work done above CP and the work expended above critical power W'_{exp}

POWER CURVES

- A rider's power curve is the maximum power a rider can constantly exert for a given amount of time.



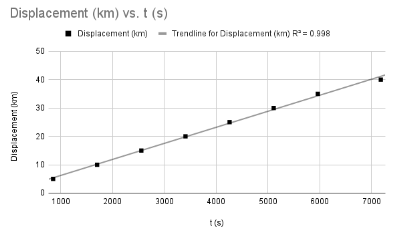
- We approximate a rider's power curve as $P = \frac{k}{t^n} + P_0$, and use regression to determine values of k and n.

SIGNIFICANCE OF SKIBA

Key Takeaway:

- **A rider should not exceed their CP, which depends on how long they are racing**
  - **the time required to recover lasts longer than the time spent above CP**

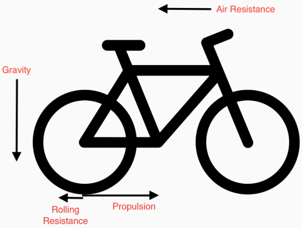
Kipchoge's Sub 2 Hour Marathon



Applied Kinematics

APPLIED KINEMATICS: WHERE DOES THE CYCLIST'S POWER INPUT GO?

- **Some intuition:**



- **This yields the equation:**

$$P_{\text{in}} = \frac{P_{\text{out}}}{\varepsilon} = \frac{P_{\text{KE}} + P_{\text{PE}} + P_{\text{AR}} + P_{\text{RR}}}{\varepsilon} \tag{3}$$

- **Note:** Each power term has its own set of derivations using $\frac{dW}{dt} = P$

POWER TERM DERIVATIONS

- ◇ Translational kinetic energy: $KE_T = \frac{1}{2}mv^2$
- ◇ Rotational kinetic energy: $KE_R = \frac{1}{2}I\omega^2$
- ◇ Gravitational potential energy: $U = mgz$
    - For the above three energies, we use $P = \frac{d}{dt}W$
- ◇ Air resistance: $F_{AR} = \frac{1}{2}C\rho Av^2$
- ◇ Rolling resistance: $F_{RR} = \frac{\mu_{RR}F_n}{r}$
    - Note that for the above two forces, we use $P = -\frac{d}{dt}\int \vec{F} \cdot d\vec{s}$

SUBSEQUENT DERIVATIONS

$$P = \frac{1}{\varepsilon}\left[\left(M_{\textsf{tot}} + \frac{3}{2}M_{\textsf{wheel}}\right)\dot{s}\ddot{s} + M_{\textsf{tot}}g\sin\theta\dot{s} + 2\frac{M_{\textsf{tot}}g\mu_{\textsf{RR}}}{r}\cos\theta\dot{s} + \alpha\dot{s}^3\right] \tag{4}$$

$$\ddot{s} = \frac{\frac{eP}{\dot{s}} - M_{\textsf{tot}}g\sin\theta - 2\frac{M_{\textsf{tot}}g\mu_{\textsf{RR}}}{r}\cos\theta - \alpha\dot{s}^2}{M_{\textsf{tot}} + \frac{3}{2}M_{\textsf{wheel}}} \tag{5}$$

——— Numerical Method ———

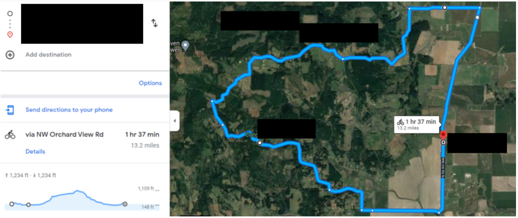
$$\dot{s}\left(t+dt\right) = \dot{s}\left(t\right) + \ddot{s}dt \tag{6}$$

$$s\left(t+dt\right) = s\left(t\right) + \dot{s}\left(t+dt\right)dt \tag{7}$$

Applied Rider Data

APPLIED RIDER DATA

- We applied our model to 8 riders on 3 courses:
  - 2021 Olympic Individual Time Trials
  - 2021 UCI World Championships ITTs
  - Le Tour de Mac



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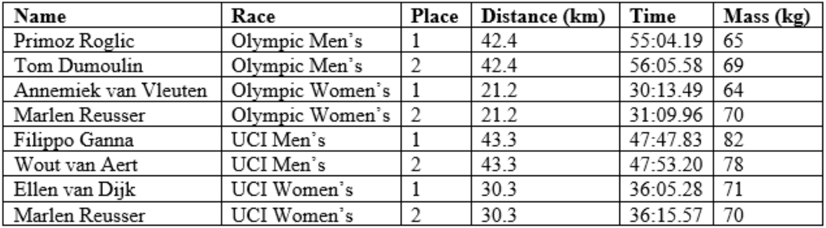
RIDER STATS



Name	**Race**	**Place**	**Distance (km)**	**Time**	**Mass (kg)**
Primoz Roglic	Olympic Men's	1	42.4	55:04.19	65
Tom Dumoulin	Olympic Men's	2	42.4	56:05.58	69
Annemiek van Vleuten	Olympic Women's	1	21.2	30:13.49	64
Marlen Reusser	Olympic Women's	2	21.2	31:09.96	70
Filippo Ganna	UCI Men's	1	43.3	47:47.83	82
Wout van Aert	UCI Men's	2	43.3	47:53.20	78
Ellen van Dijk	UCI Women's	1	30.3	36:05.28	71
Marlen Reusser	UCI Women's	2	30.3	36:15.57	70

MODEL RESULTS

Estimated Power Output

Name	Race	Place	Distance (km)	Time	Mass (kg)	Est. Power Output (W)
Primoz Roglic	Olympic Men's	1	42.4	55:04.19	65	435.48
Tom Dumoulin	Olympic Men's	2	42.4	56:05.58	69	462.04
Annemiek van Vleuten	Olympic Women's	1	21.2	30:13.49	64	384.79
Marlen Reusser	Olympic Women's	2	21.2	31:09.96	70	420.42
Filippo Ganna	UCI Men's	1	43.3	47:47.83	82	551.72
Wout van Aert	UCI Men's	2	43.3	47:53.20	78	524.77
Ellen van Dijk	UCI Women's	1	30.3	36:05.28	71	424.38
Marlen Reusser	UCI Women's	2	30.3	36:15.57	70	418.34

Model Times vs. Race Times

Name	Race	Place	Time	Predicted Time
Primoz Roglic	Olympic Men's	1	55:04.19	56:38.10
Tom Dumoulin	Olympic Men's	2	56:05.58	55:42.70
Annemiek van Vleuten	Olympic Women's	1	30:13.49	30:08.50
Marlen Reusser	Olympic Women's	2	31:09.96	29:25.40
Filippo Ganna	UCI Men's	1	47:47.83	49:24.10
Wout van Aert	UCI Men's	2	47:53.20	50:08.30
Ellen van Dijk	UCI Women's	1	36:05.28	37:53.70
Marlen Reusser	UCI Women's	2	36:15.57	38:08.30

Note: Predicted times were found numerically from model

Team Time Trial Optimization

TEAM TIME TRIAL OPTIMIZATION

- ◇ In a team time trial, a team of six cyclists compete together, trying to minimize the fourth-place rider's time.
- ◇ Unlike an individual time trial, riders can draft off each other.
  - Riders can have a faster velocity for a given power level since air resistance would be decreased.
  - The most efficient drafting formation would be when riders are directly behind each other.
  - In general, riders should maintain order of power levels, but occasionally the fifth and sixth riders can be ahead to allow the first four riders to have a faster time.

Closing Remarks

REMARKS

- **Assumptions of our Model**
  - Constant Air density and no wind
    - With wind, P_{AR} would become:

$$P_{AR} = \alpha ||\vec{v}_{wind} - \vec{v}_{bike}||^2 ||\vec{v}_{bike}|| \tag{8}$$

  - Perfect traction
  - Same mass of bikes between riders
- **Key Exclusion in Our Model:**
  - Curvature of courses:

$$v_{\text{max}} = \sqrt{\frac{1}{\kappa}\mu_{\text{RR}} g \cos\theta \tan\phi} \tag{9}$$

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