Computationally Modeling Compton Scattering Using Monte Carlo Methods

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Radiation Therapy

A popular method of treatment:
- Effective against cancer
- Much shorter treatment than chemotherapy
- Eliminate cancerous tissues from the human body, while leaving healthy tissue in tact.

Technological improvements under way:
- More precise and versatile radiation equipment
- More sophisticated software for treatment plans
- Greater understanding of interactions and simulations needed.

Theory

In radiation therapy, the main interactions are:
- Photoelectric effect- low energy
- Pair production- high energy
- Compton Scattering – most common

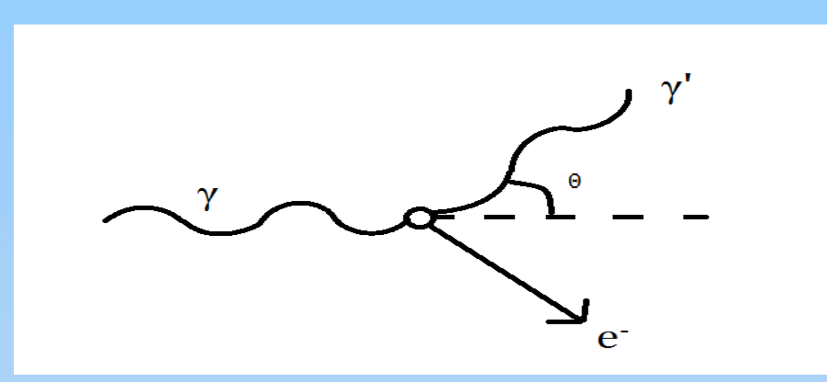

Figure 1: A diagram of Compton scattering

Compton scattering

- The incoming photon collides with an electron at rest.
- Total energy and momentum are conserved.
- Energy deposited to electron is dependent on scattering angle:

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\Theta)$$

Monte Carlo Method

The Monte Carlo method is a computational tool useful for modeling indeterministic situations like Compton scattering.
- Based on the probability of the interaction
- Random generation of trial data

Differential Cross-section

- Describes distribution of probability (dσ) over the solid angle (dΩ).
- Differential cross-section for Compton scattering is Klein-Nishina formula [2]:

$$F_{KN} = \left[\frac{1}{1+\alpha(1-\cos\Theta)}\right]^2 \left[1+\frac{\alpha^2(1-\cos\Theta)^2}{(1+\alpha(1-\cos\Theta))(1+\cos^2\Theta)}\right]$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{r_0^2}{2}\right)(1+\cos^2\Theta)F_{KN}$$

Abstract

Radiation therapy is becoming a popular method of cancer treatment due to its effectiveness and precision. Improvements in radiation therapy treatments arise from an increased understanding of the interactions between radiation and matter, where Compton scattering is the primary interaction. The main objective of this research project is to model Compton scattering using the program Matlab, to better understand energetic and spatial distribution of scattering events. Monte Carlo Methods and the Klein-Nishina formula were used to simulate the interactions within a material. It was found that a simulation with a higher number of events more realistically reflects the probabilistic nature of this interaction and the range of possible scattering outcomes, and shows the way in which energy would be deposited when a patient is irradiated with a large number of photons. The effect of the initial photon energy and cut-off interaction energy in the simulation are also explored.

Methods

To computationally model Compton Scattering, the simulation can be broken down into different sections:

1. Angle Portion
   - Position of event
   - Angle of outgoing photon
   - Probability of event occurring

2. Energy Portion
   - The photon will continue through the material until the photon no longer has enough energy to interact.
   - At each interaction the energy deposited at each spot in the lattice will be recorded.
   - A new photon is generated when there is no longer energy from the previous photon.

3. Full Simulation

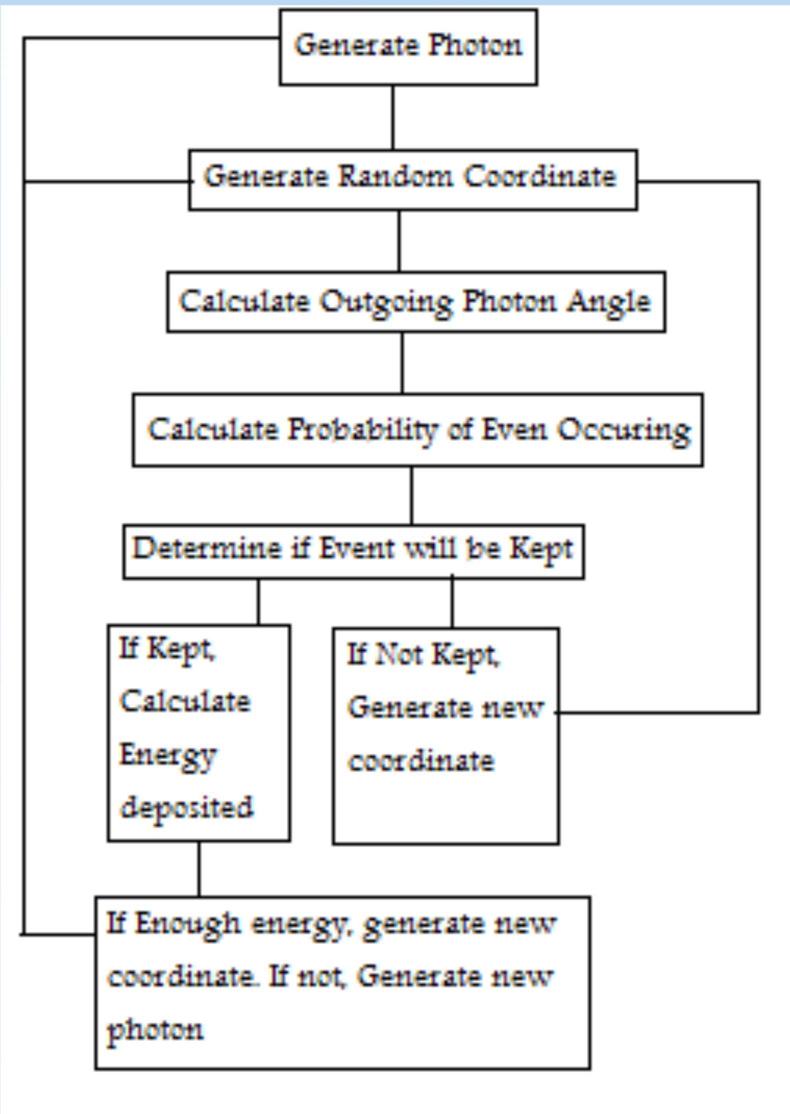

Figure 2. Flow chart showing the simulation steps.

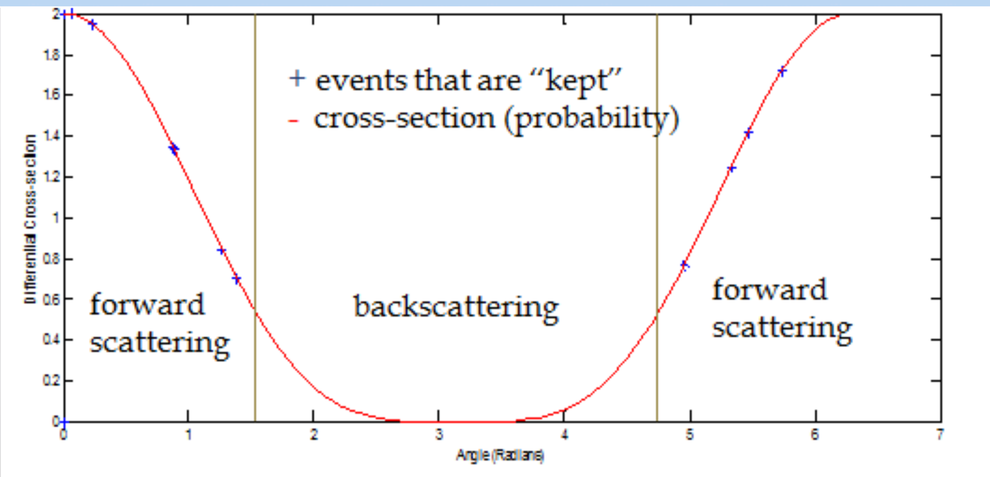

Figure 3: Generation of 6MeV photons, showing the scattering angles of 30 photons.

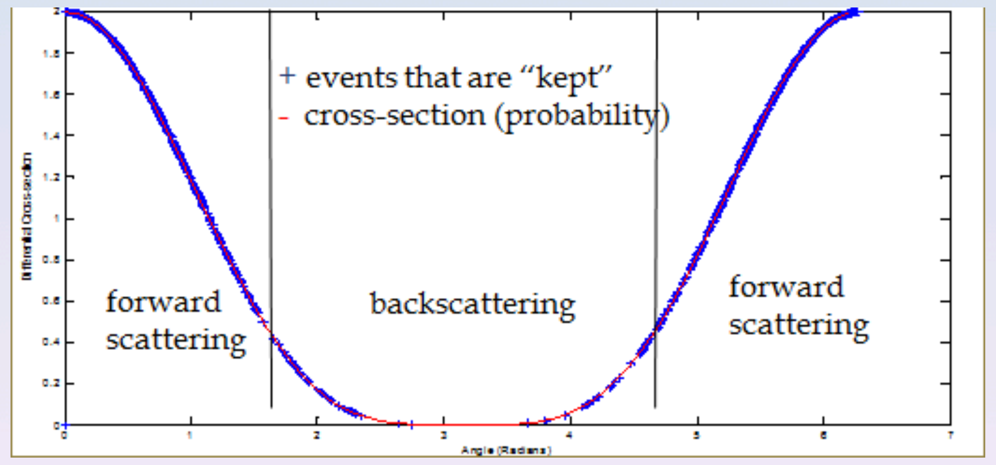

Figure 4: Generation of 6MeV photons, showing the scattering angles of 3000 photons.

Results and Analysis

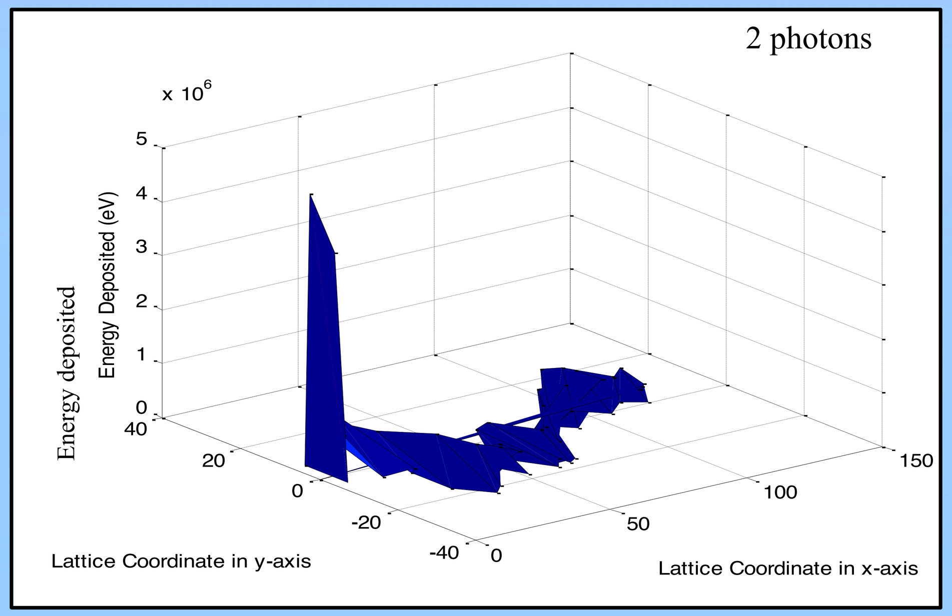

Figure 5: Two photons scattering through the material, with an initial energy of 6 MeV.

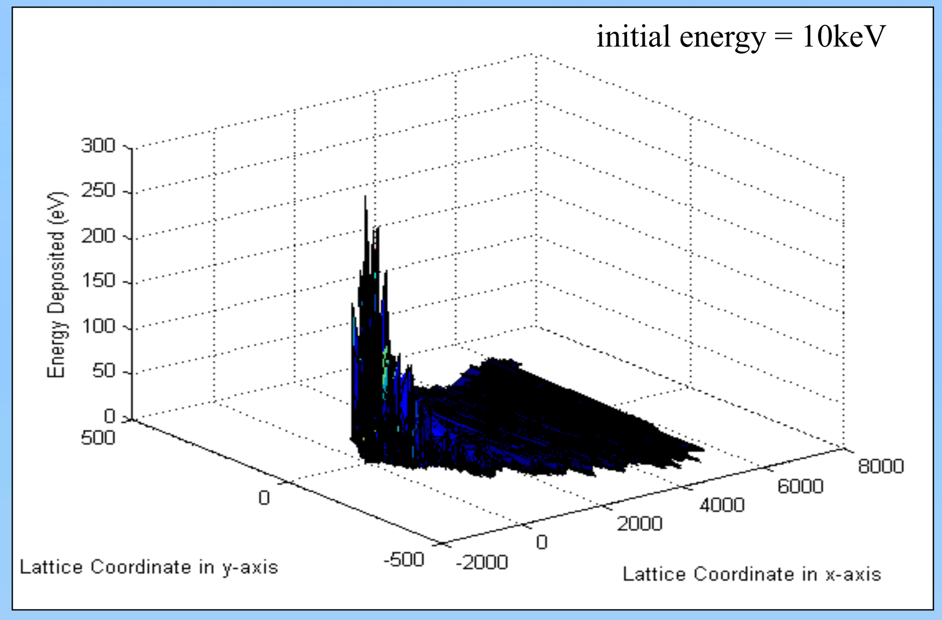

Figure 6: 10 photons scattering through the material, with an initial energy of 10 keV.

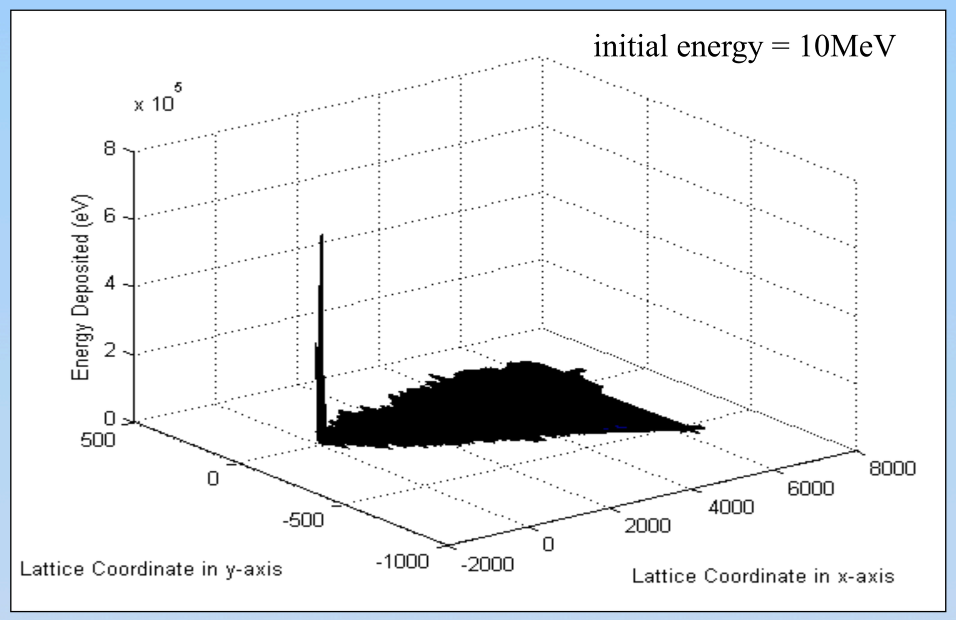

Figure 7: 10 photons scattering through the material, with an initial energy of 10 keV.

Conclusion

- Full simulation of Compton Scattering successfully generated.
- Full simulation agreed with expectations:
  - More photons yields a more realistic simulation.
  - Higher initial energy yields higher initial interactions and a higher percentage of the total energy deposited near the surface.
  - Lower cut-off energy yields greater distance irradiated within the material.

References

[1]Harold Johns, John Cunningham, *The Physics of Radiology,* (Charles C. Thomas, University of Michigan, 1983).
[2]Multicare,http://staging.multicare.org/home/cancer-care-technology, 2011
[3]Peter R. Fontana, "Atomic Radiative Processes", *Pure and Applied Physics,* Vol. 42. (Academic Press, New York, 1982).

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