5-2018

Effects of Tension on Resonant Frequencies of Strings

Blake Burnett
Linfield College

Recommended Citation

Burnett, Blake, "Effects of Tension on Resonant Frequencies of Strings" (2018). *Senior Theses*. 35.
https://digitalcommons.linfield.edu/physstud_theses/35

Effects of Tension on Resonant Frequencies of Strings

Blake Burnett

A THESIS
Submitted in partial fulfillment of the requirements for the
degree of Bachelor of Science

Department of Physics
Linfield College
McMinnville, OR
May 2018

THESIS COPYRIGHT PERMISSIONS

Please read this document carefully before signing. If you have questions about any of these permissions, please contact the DigitalCommons Coordinator.

Title of the Thesis:

Effects of Tension on Resonant Frequencies of Strings

Author's Name: (Last name, first name)

Burnett, Blake

Advisor's Name

Jennifer Heath

DigitalCommons@Linfield (DC@L) is our web-based, open access-compliant institutional repository for digital content produced by Linfield faculty, students, staff, and their collaborators. It is a permanent archive. By placing your thesis in DC@L, it will be discoverable via Google Scholar and other search engines. Materials that are located in DC@L are freely accessible to the world; however, your copyright protects against unauthorized use of the content. Although you have certain rights and privileges with your copyright, there are also responsibilities. Please review the following statements and identify that you have read them by signing below. Some departments may choose to protect the work of their students because of continuing research. In these cases, the project is still posted in the repository but content will only be accessible by individuals who are part of the Linfield community.

CHOOSE THE STATEMENT BELOW THAT DEFINES HOW YOU WANT TO SHARE YOUR THESIS. THE FIRST STATEMENT PROVIDES THE MOST ACCESS TO YOUR WORK; THE LAST STATEMENT PROVIDES THE LEAST ACCESS. CHOOSE ONLY ONE STATEMENT.

X I **agree** to make my thesis available to the Linfield College community and to the larger scholarly community upon its deposit in our permanent digital archive, DigitalCommons@Linfield, or its successor technology. My thesis will also be available in print at Nicholson Library and can be shared via interlibrary loan.

OR

_____ I **agree** to make my thesis available **only** to the Linfield College community upon its deposit in our permanent digital archive, DigitalCommons@Linfield, or its successor technology. My thesis will also be available in print at Nicholson Library and can be shared via interlibrary loan.

OR

_____ I **agree** to make my thesis available in print at Nicholson Library, including access for interlibrary loan.

OR

_____ I **agree** to make my thesis available in print at Nicholson Library only.

Updated April 25, 2018

NOTICE OF ORIGINAL WORK AND USE OF COPYRIGHT-PROTECTED MATERIALS:

If your work includes images that are not original works by you, you must include permissions from the original content provider or the images will not be included in the repository. If your work includes videos, music, data sets, or other accompanying material that is not original work by you, the same copyright stipulations apply. If your work includes interviews, you must include a statement that you have the permission from the interviewees to make their interviews public. For information about obtaining permissions and sample forms, see https://copyright.columbia.edu/basics/permissions-and-licensing.html.

NOTICE OF APPROVAL TO USE HUMAN OR ANIMAL SUBJECTS:

If your research includes human subjects, you must include a letter of approval from the Linfield Institutional Review Board (IRB); see https://www.linfield.edu/faculty/irb.html for more information. If your research includes animal subjects, you must include a letter of approval from the Linfield Animal Care & Use Committee.

NOTICE OF SUBMITTED WORK AS POTENTIALLY CONSTITUTING AN EDUCATIONAL RECORD UNDER FERPA:

Under FERPA (20 U.S.C. § 1232g), this work may constitute an educational record. By signing below, you acknowledge this fact and expressly consent to the use of this work according to the terms of this agreement.

BY SIGNING THIS FORM, I ACKNOWLEDGE THAT ALL WORK CONTAINED IN THIS PAPER IS ORIGINAL WORK BY ME OR INCLUDES APPROPRIATE CITATIONS AND/OR PERMISSIONS WHEN CITING OR INCLUDING EXCERPTS OF WORK(S) BY OTHERS.

IF APPLICABLE, I HAVE INCLUDED AN APPROVAL LETTER FROM THE IRB TO USE HUMAN SUBJECTS OR FROM ANIMAL CARE & USE TO USE ANIMAL SUBJECTS.

Signature *Signature redacted* Date 5/16/2018

Printed Name Blake Burnett

Approved by Faculty Advisor *Signature redacted* Date 5/16/2018

Thesis Acceptance

Linfield College

Thesis title: Effects of Tension on Resonant Frequencies of Strings

Submitted by: Blake Burnett

Date submitted: April 5, 2018

Signature redacted

Thesis Advisor: Dr. Jennifer Heath

Physics Department:

Signature redacted

Dr. William Mackie

Physics Department: *Signature redacted*

Dr. Tianbao Xie

Abstract

This project tests and explores resonance of strings. Since all materials and mechanisms are affected by vibrations, it is important to know the frequencies at which resonance occurs. To explore this subject, strings were used as a model material to test the effect tension has on resonance. The fundamental frequencies and the corresponding modes of resonance were used to analyze the data. The results of this experiment show that increasing tension on a string increases its resonance frequency. Understanding the physics behind resonance frequency allows systems to be designed to take advantage of resonance properties, or to avoid resonance where it can be destructive.

Table of Contents

List of Figures

1. Background

Vibrations and resonance are what helps make up the surrounding materials and mechanisms. Vibrations can be applied to any area from the universal level to the subatomic level. There are many different forms of vibration that can be explored. Vibrations are used to promote harmony, like pianos and planetary orbits. In contrast, it is also known in destruction, such as singing to shatter a wine glass or in the collapse of Tacoma Narrows Bridge. These few examples show the importance of exploring resonance, thus making it not only relevant, but crucial information in many subjects.

1.1 Vibration Background

To understand vibrations and their applications, the areas of natural frequency and resonance should be defined. Natural frequency is the frequency, measured in Hertz, at which a system oscillates when not subjected to a continuous or repeated external force. The external forces can either be a driving or an opposing force. When an external force is applied it will most often result in dampened oscillations unless the natural frequency is approached. When forced frequencies match the natural frequency it is called resonance. More specifically, resonance is when an external force drives another system to oscillate with the greatest amplitude at specific frequencies.

Since most objects are in motion it can be assumed that those things have their own natural frequency, making it a subject of study throughout many different fields, each using their own set of parameters resulting in multiple definitions of resonance. The different categories that specifically address this subject can include: electrical, chemical, optical, orbital, acoustic, mechanical, and atomic. For this thesis, the focus will be on the principles of mechanical and acoustic resonance of strings.

1.2 Mechanical Resonance

Mechanical resonance is most often related to examples including an oscillating spring or a swinging pendulum. One way to visualize mechanical resonance is to imagine someone swinging on a swing set. If someone were to push someone else on a swing set from one direction, only at the peak will the swing motion increase relatively quickly, even if the applied force was small. On the other hand, if the direction of the applied force was out of sync, or continuous in one direction with the swing then the expected result would be a decrease in motion and amplitude. This concept is used similarly when applied to string resonance.

String resonance is where a length of wire shows distinct waves, forming nodes and antinodes. These waves can also be applied to hollow columns of air. A visual representation of string resonance is shown by Figure 1.1.

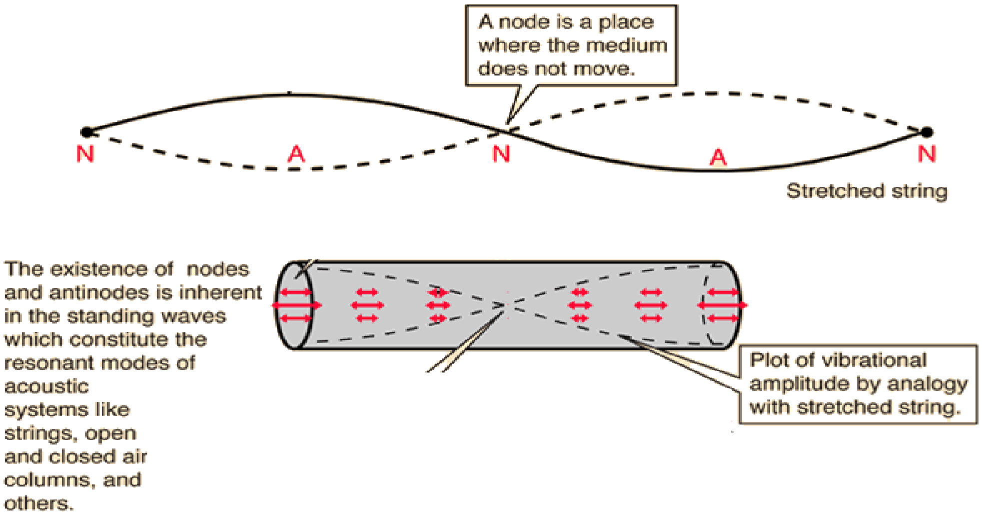

Figure 1.1: Definition of Resonance. Explains resonance of both open ended and closed ended resonance. Nodes are marked by N. Replicated from HyperPhysics Concepts[1].

As resonance increases the number of nodes increases, the frequency at where these multiples occur are called modes of resonance. Different shapes, lengths, and the material itself can change where the modes of resonance occur. Figure 1.1 compares a tube with both ends open versus a tube with one closed end. The modes of resonance were altered from this one change.

1.3 Applications of Mechanical Resonance

Mechanical and string resonance are used in musical instruments, clocks, and even buildings. Figure 1.2 shows how mechanical resonance is avoided by dampening movement of a skyscraper. The large mass acts as a dampening counterbalance to the skyscraper. Since the hanging mass is free to move about the building it can sway in an opposing direction to counteract the buildings movement. In theory, the momentum of the building plus the free hanging mass will result in a total momentum of zero. This is just one example of the various ways to avoid resonance in buildings in a constantly changing and developing field. It would be useful to be able to predict where resonance would occur. Exploring string resonance would be a helpful start on how to predict modes of resonance.

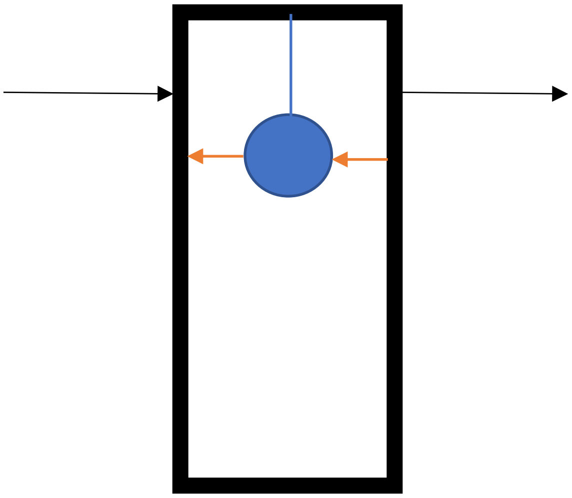

Figure 1.2: An example of a tower that uses a large pendulum high in a skyscraper to dampen swaying of the building. As momentum of building moves one direction, the hanging mass's momentum moves in the opposing direction. The addition of momentums goes to zero.

2.Theory

Finding the resonance of a material or system can often times be useful. Using a state of resonance can determine the stability of a system. Frequency can be modeled by mathematical equations and used to formulate figures and graphs.

2.1 Finding resonance

Locating the resonant frequency is done by applying a force or energy to a system where its periodic motion is matched by the timing and direction of force or energy. If the timing or direction does not match the periodic motion, then there will be opposing forces causing the mechanisms to dampen. Most of the time, objects are in a dampened state, therefore they are in a motionless state. When at resonance it is expected that a system would be driven to peaks, moving with great energy and in some cases cause destruction. Equation 1 is often used to find simple mechanical resonance.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{1}$$

Where k is the spring constant and m the is mass.

When frequency is graphed against amplitude, the resonant frequencies would be where the amplitude is at its peaks. The rest of the graph would have very little amplitude. An alternate way to compare resonance would be to plot frequency against transmissibility shown in Figure 2.1 below.

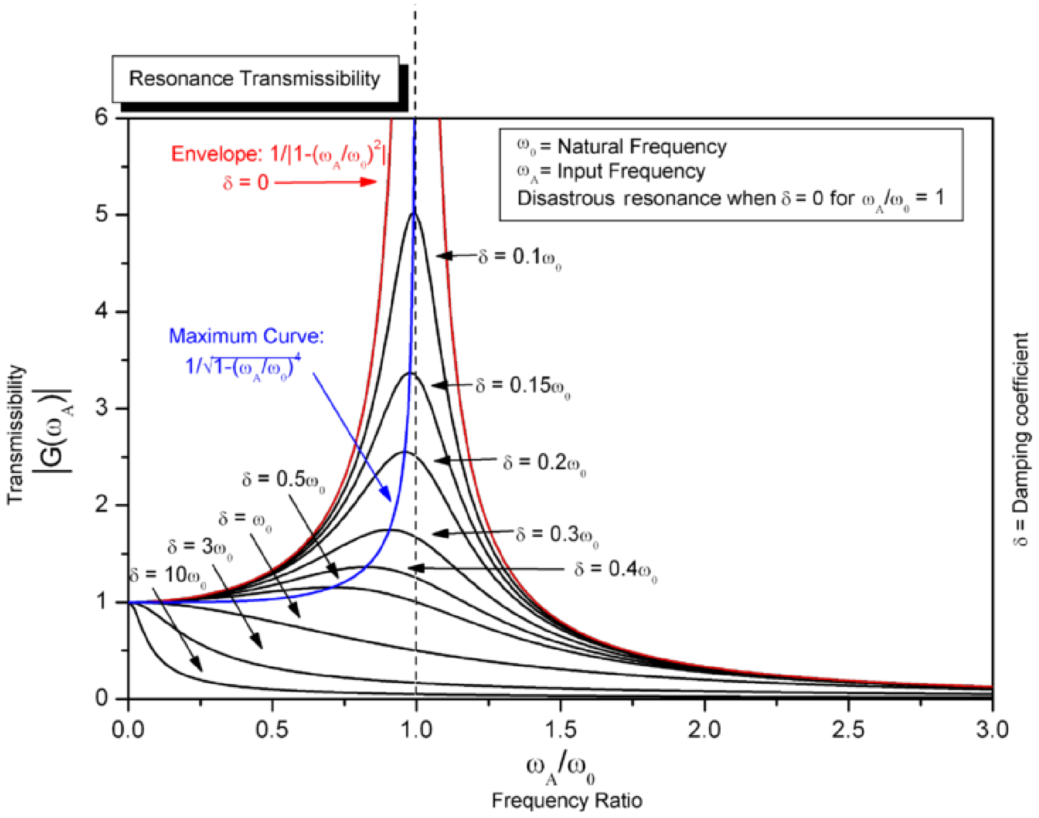

Figure 2.1: A graphical representation of resonant frequency comparing transmissibility versus frequency. Peaks where input frequency ω_A is equal to natural frequency ω_0. Transmissibility is where the frequency can move the best without resistance [2,3].

A good intuition for what resonance would look like can be gathered from Figure 2.1 and applied to various fields including the resonance of strings. Referring to Figure 2.1, resonance is realized by where the amplitude is at its greatest. Transmissibility, the x-axis, is the ability of the material to conduct a certain frequency ratio. The transmissibility is at its greatest when the frequency ratio is at one. At this point, the frequency ratio is where the input frequency and resonant frequency are equal. A determining factor for the height of the transmissibility peaks is the damping coefficient. The dampening coefficient is determined by outside factors unrelated to the interference that occurs at frequencies away from resonance causing the transmissibility to approach zero.

2.2 String Resonance

Strings have different resonance identities when compared to a simple mass on a spring or a simple pendulum. The variables in Equation 1 change from a spring constant k and hanging mass m when analyzing string vibration. First, the wave velocity must be found using Equation 2:

$$v = f \cdot \lambda \tag{2}$$

f is the frequency of the wire and λ is the wavelength. To identify λ, the length of the span of wire is divided by the number of nodes at a particular resonant frequency. The wavelength can be calculated by Equation 3:

$$\lambda_n = \frac{2L}{n} \quad n = 1,2,3,\ldots \tag{3}$$

L is the total length of the span and n is the number of nodes. Since the wave is on a string the following Equation 4 will be used to calculate the wave speed:

$$v = \sqrt{\frac{T}{m/L}} \tag{4}$$

In Equation 4, v is the wave speed, T is the tension on the wire, and m/l is the mass per unit length. Equation 4 gives the velocity of a string stretched across a span. The velocity is determined by the tension T and mass per unit length m/L. Now the wave relationship can be applied to the stretched string that will then produce a standing wave. Strings have multiple modes of resonance called harmonics, the area where the strings vibrate most easily.

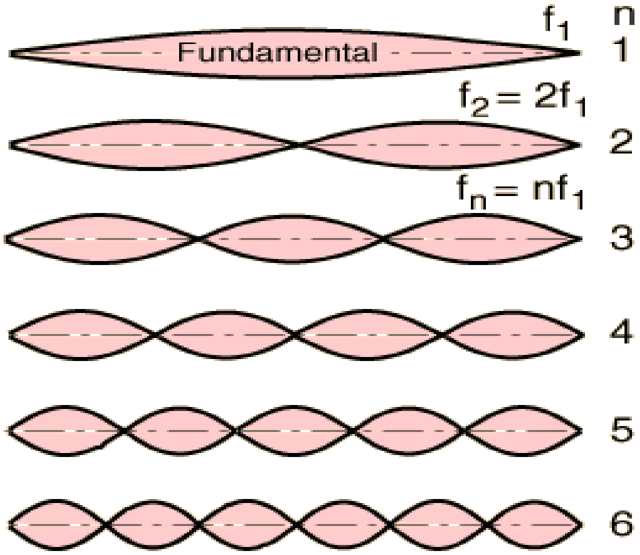

Figure 2.2: Modes of resonance called harmonics. N represents the number of nodes on the string at a given resonant frequency. [1]

The Fundamental frequency is defined as the lowest frequency mode and can be calculated by using the velocity of a string and dividing it by half of the string length as shown in Equation 3

$$f_n = \frac{n\sqrt{\frac{T}{\left(\frac{m}{l}\right)}}}{2L} \tag{5}$$

Where T is the tension m/L is the strings mass per unit length and L is string length and n is the number of nodes. Equation 5 can be used to determine the resonant frequency far all modes of resonance; knowing where resonance will occur can be very useful in its applications.

3. Experiment

Every material and system has a resonant frequency. The resonant frequency is the natural frequency of vibration of a material or system that experiences no dampening forces. This frequency is parameterized by many factors such as size, shape, composition and tension. In this experiment, cotton aluminum, and steel strings were used. A system made of a certain material and length has a predictable resonant frequency.

3.1 Materials

In order to set up and explore resonance some materials were gathered. The materials included: strings of different densities and materials (cotton, aluminum, steel), a speaker capable of attaching to a wire, a function generator with variable frequencies, an amplifier, a stand and connectable frictionless pulley, an assortment of masses (30 to 100 grams) with an attachable hook for the string. These materials were used for Section 3.2.

3.2 Set up

This experiment was performed and set up according to Figure 3.1. This was done by first connecting the amplifier and function generator to a power source, which was a standard American 120 Volt wall plug. Since function generators typically operate at low voltages, they lack the minimum power to drive the speaker. The amplifier additionally allowed for greater control over the amplitude of wire movement. Once the function generator, amplifier, and speaker are all connected, one of the variable strings can now be connected to the speaker. The speaker had a small metal clip attached to the center of the speaker that was clamped to the string. The string was then strung across to a stand that had an assumed frictionless pulley

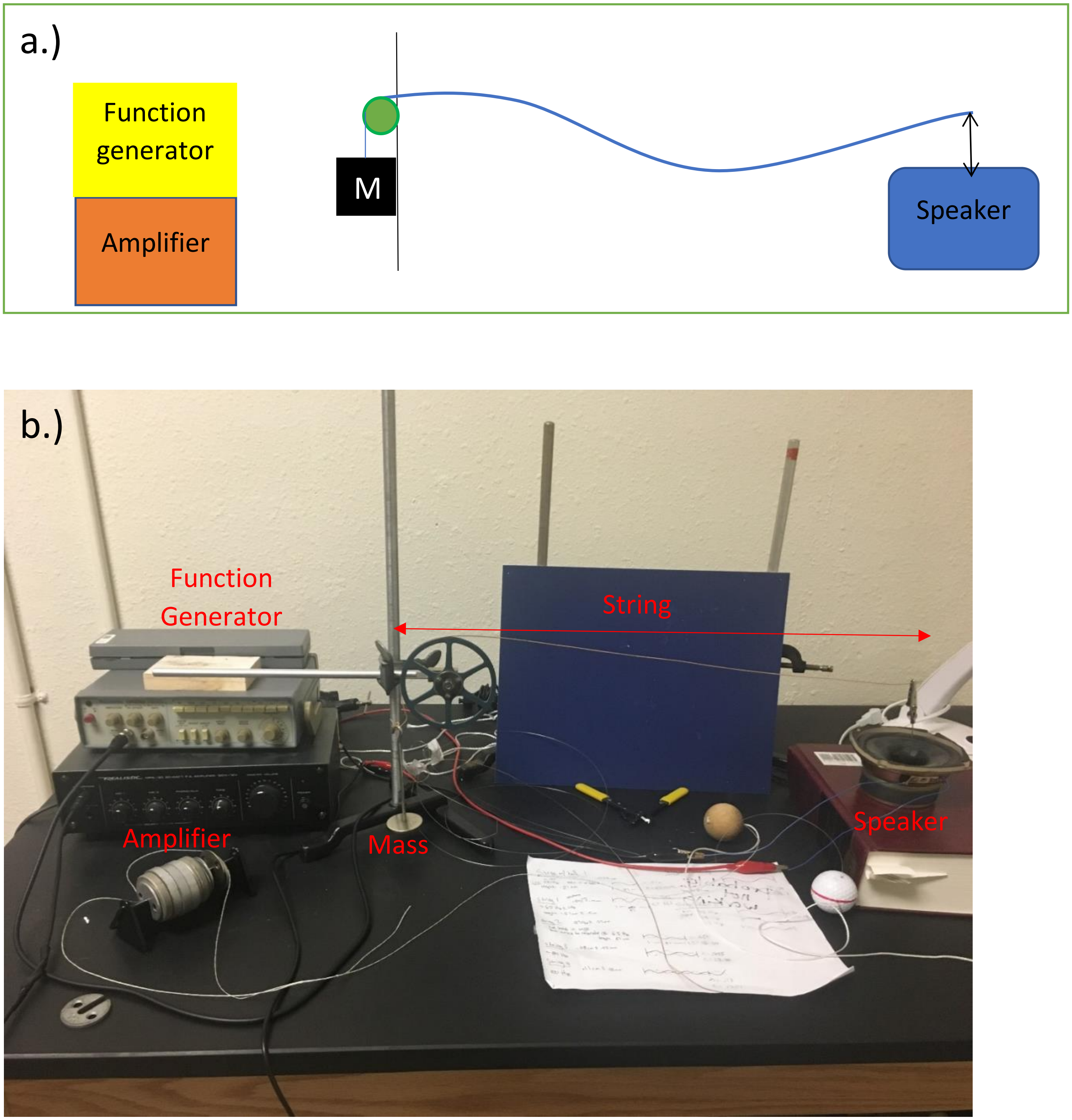

Figure 3.1: Set up of experiment. Function generator, amplifier, and speaker connected by wires. Hanging mass displayed as M and string displayed as wave spanning from frictionless wheel to the speaker. (b) Actual set up of experiment.

attached. A mass was connected to ensure a constant tension on the string. Now the experiment was properly prepared, and the experiments described in Section 3.3 could be conducted.

3.3 Experiment

This experiment consists of two sub sections, varying strings and varying tension. These experiments help to better understand wave and resonance behavior and the effects each of the variables will produce. Each section was executed by finding resonance through the scanning of frequencies which will be further explained in the following sections.

3.3.1 Varying Modes

The experiment used steel guitar wires, an aluminum wire, and cotton strings, attached to a function generator and amplifier. A string was strung in a straight plane from a frictionless pulley, with a 50-gram mass applied to the end, to the opposing end and attached to the speaker. The string had a span from the pulley to the speaker of 51 cm (uncertainty ±0.5cm). The frequencies were then scanned using the function generator from low to high frequency which revealed the resonance for each strings parameter. The speaker served as a one-dimensional function generator because it moved the wire along a single vertical axis. The resonant frequency was clearly shown by distinct waves revealing nodes along the wire. The frequencies were scanned from high to low frequency noting where different nodes of resonance occurred. Then the scanning of frequencies for the same wire was then repeated in the opposite direction, from low to high frequencies. This process was done to increase accuracy the experiment. The next string replaced the previous and the experiment was repeated for each of the varying strings.

3.3.2 Varying Tension

The experiment consisted of hanging varying masses on one end of the string, in 10-gram (0.01 kg) intervals, then observed and recorded how resonance was affected. The frequencies were scanned carefully and similarly to section 3.3.1, when a stable mode of resonance was found it was kept for the entirety of the experiment for that string. Note that a different mode of resonance could be used for a different string because the affects of tension on resonance are not affected by the number of nodes. Tension was added to the string by placing more mass at the end in 10-gram increments. It was difficult to test higher tensions because of limitations in equipment. The higher tensions, above 100 grams, pulled too hard on the speaker and risked ripping and breaking it or caused significant distortion in the length of the string. This is source of error in the length of the string. The movement in the x-direction will change the resonant frequency unintentionally because string length has a direct effect on resonance. In hopes to gain more abundant and accurate data points, increasing the tension by a smaller interval is possible. This slight alteration would not significantly change the data that was already taken because the interval of change would be insignificant. Already, the change in frequency between the changes in resonance were slight changes, measurable by a few Hertz.

3.4 Measurements

Getting accurate and dependable data is achieved by using reliable measuring instruments. Various instruments were used in the measuring process each yielded different accuracy and precision. The mass of the wire was measured by a triple beam scale which gave an uncertainty ±0.01 grams. Each wire was coiled and tied so that the complete string was being measured. The length of each string and the length of the span were measured using a meter stick giving an uncertainty of ±0.5 cm. The radius of the strings was measured using calipers giving an uncertainty of ±0.02 mm. Recording the resonant frequencies displayed by the function generator was done so with an uncertainty ±0.5 Hz (Hertz). The precision of the function generator could be increased by using a digital displayed function generator of frequency rather than a dial display.

While the experiment was in motion and string oscillations were occurring, there were additional sources of error that need to be discussed. The error can be found at the two endpoints. At one end of the experiment, the oscillating speaker allows the string to experience both horizontal and vertical forces. Ideally, the string only moves along the vertical y-axis, but since the piece connecting the speaker and string is not completely rigid it results in movement in the horizontal x-direction. The string moving in more than one plane causes some misleading and hectic vibrations increasing the source of error. On the other end of the string, where the mass, string, and pulley interact, there is also a source of error. When the string is in motion there is a slight back and forth tugging action. The tugging action makes a slight change in length of the string. The change in length directly effects the resonant frequency of the string according to all equations dealing with vibrations. Since the length variation is only subtle we can still use the experiment to gather data with small amounts of error.

4. Results and Analysis

The data collected from the varying of nodes and varying tensions on many different strings results in the following sections. The comparison between the two variations will then be summarized and related.

4.1 Variable modes

The first set of experimental data is provided by varying string material and string density. The results are reported in Figure 4.1 in terms of wave speed versus density of wire (also referred to as mass per unit length). Data was originally gathered in terms of frequency and number of nodes. From this information, wave speed can be calculated using Equation 1 found in the Section 2.

According to Equation 3, v is the wave speed, T is the tension on the wire (which is constant for this experiment), and m/l is the mass per unit length. The data used to plot Figure 4.1 is the average wave speed for each string, computed by Equation 3. Since the wave speed is ideally constant for its string the average value gives the most accurate value. The values on Figure 4.1 follow the theoretical line of a square root function showing the decreasing trend of wave speed as mass density increases.

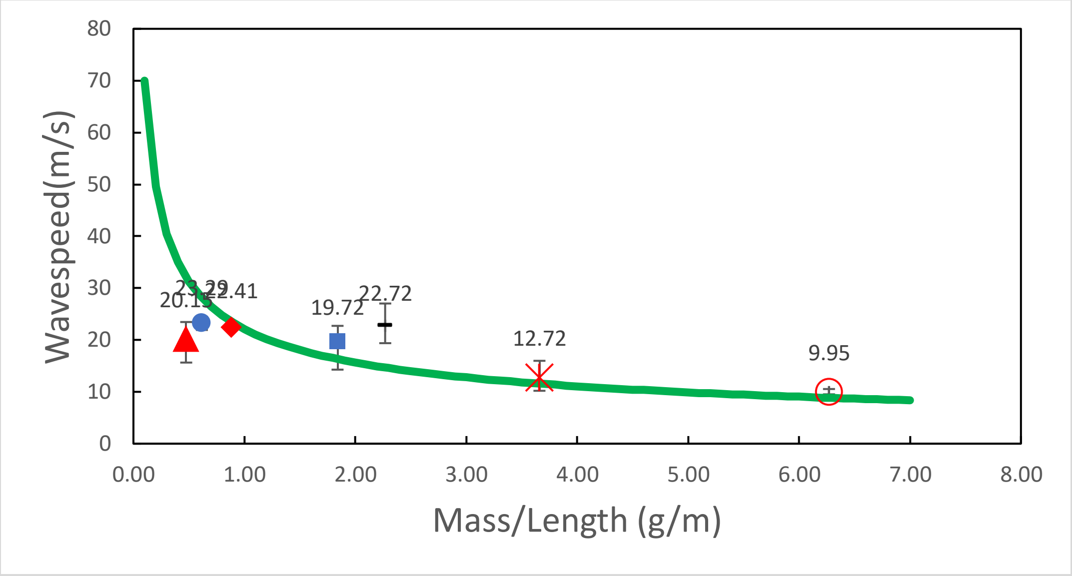

Figure 4.1: Density of string or wire (grams per meter) against average Wavespeed (meters per second). Red symbols represent guitar strings, blue symbols are cotton strings, the black line is the aluminum wire. The green line is the theoretical trend. Error bars created from the distance away from average value.

Now that the behavior of wave speed and mass per length show a square root relationship, that information can be used to further understand resonance and its variables.

Finding the fundamental frequency for some of the wires was difficult due to the restraints of the experimental set up. For example, the densest string, guitar four, would only produce a minimum of four nodes. The relationship between the increase in frequency and the resonant frequencies for different wires is shown by Figure 4.2.

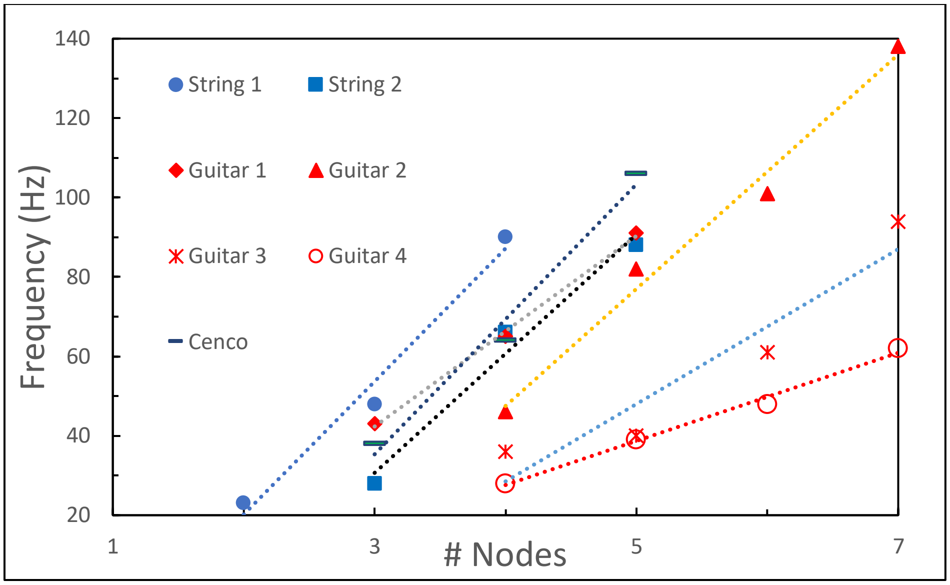

Figure 4.2: Shows at which frequency nodes were produced for each wire. Each wire shows a linear trend for resonant frequencies. The string with the greatest density, guitar 4, has the lowest frequency and the greatest number of nodes.

A linear relationship helps to show that resonant frequencies can be modelled and predicted. The best way to model at which frequency a wire will resonate is with Equation 5, located in Section 3. This allows for the computation of the resonant frequency at its varying modes. The slopes of the lines are all linearly increasing but each have a unique slope. The strings with greater mass densities resonate at lower frequencies and higher number of nodes.

4.2 Varying Tension

The second part of the experiment used a string with a constant node and the tension for each string was varied. Doing this tested the reliability of how tension affects a strings resonance. For String 1 and String 2 as well as Guitar 1, Guitar 2, the data was reported where 3 nodes were present. Guitar 3 data taken at 4 nodes and Guitar 4 taken at 5 nodes. The wires could not be taken at the same number of nodes due to the limitations in the set up and equipment referred to in Section 3.3 and Section 3.4. The higher tensions pulled too hard on the speaker and risked ripping and breaking it. It is possible to put the strings at different nodes on the same graph because for each independent string, the number of nodes was kept constant for the duration of the experiment. In addition, the purpose of varying tension is to show the trend of resonance compared to changing tension. Which leads to the results depicted by Figure 4.3.

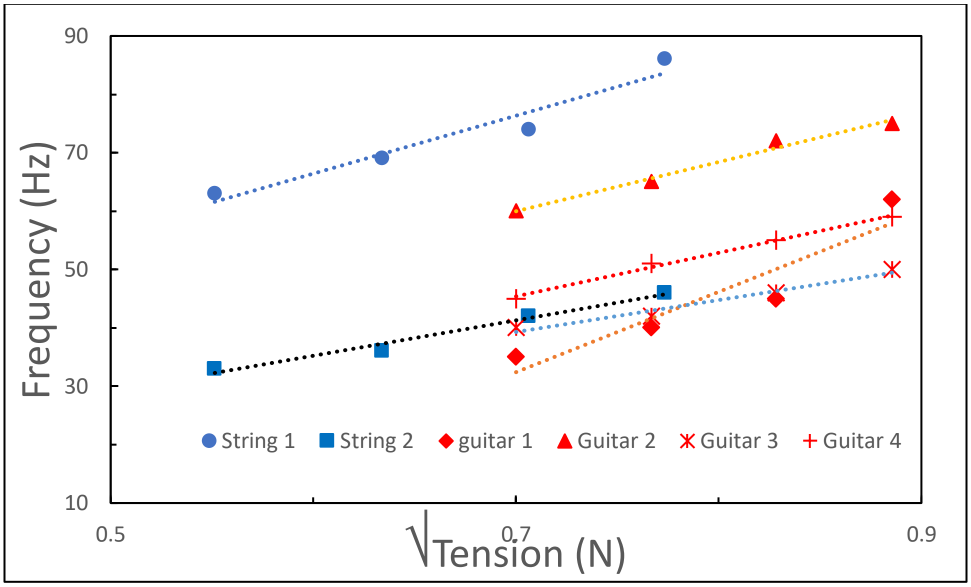

Figure 4.3: Shows the frequency at a particular mode of resonance and the square root value of tension (Newton) for wires. A linear relationship between frequency and tension can be observed by the each of the wires.

The square root of tension was taken to create the horizontal x-axis. The square root was taken because the force of tension in the wire is related to the wave speed and wire density. The added tension causes the resonance to increase because it takes a greater wave speed to move the wire. This means that tension has a direct effect on the frequency of resonance and proficient results could be gathered.

4.3 Discussion

The data collected helps show the resonant frequency of wire as predictable. Many of the relationships are linear which makes for good expectedness and calculations. Figure 4.1 shows that the velocity of the guitar wires in general were similar. It could be possible to ask how this may have been accomplished and if they are engineered to resonate at similar wave speeds. The addition of tension on a string showed dampening affect on string amplitude and an increased resonant frequency.

Data was found where some strings began to repeat low frequency modes at higher frequencies. This data was not included in analysis because it occurred very rarely and infrequently making it inconsistent. This information could be worth further investigation in the future. This could help to establish a more specific relationship in the resonating modes, which would extend predictability.

5. Conclusion

The resonant frequency is often a substantial part in the stability of a broad spectrum of applications. This experiment has explored and revealed that tension has an effect on resonance that can be predictable. The data shows resonance of waves on a string are measurable. Through the experimental process and analysis, dependence of resonance on tension agrees with the theory that resonance is affected by tension. It is clear in this experiment that added tension decreases the amplitude of the wave on a string. Dependence on mass density is less easily understood. Further exploration considering mass density of wires would be interesting and useful information. Going forward, adding tension or compression to other materials, such as solid objects, would be an effective way to increase knowledge and usefulness to this thesis. This would consist of finding new ways to vibrate materials of different shapes and sizes in the drive to find effects of resonance.

6. Acknowledgements

To conclude this document, I would like to thank the Physics Department at Linfield College. I would also like to thank the professors on my Thesis Committee who guided me through every step of the process, providing support and information to end with a fantastic learning experience. I would like to directly thank Dr. Heath for the development of this project, Dr. Mackie for his expertise in the fine tuning of the experiment, Dr. Xie for his assistance in the shop and other various areas of the completion of this thesis. Lastly, tank you to all my peers and friends for the time and support throughout the endeavor to become a better scientist.

7. References

[1] Nave, C. R. (2000, August). HyperPhysics. Retrieved from http://hyperphysics.phy-astr.gsu.edu/hbase/index.html

[2] Ghatak, A. K. (2005). *Optics* (3rd ed.). New Dehli: McGraw-Hill. p. 6.10.

[3] Ogata, K. (2004). *System Dynamics*(4thed.). United States: Pearson Education. P. 617.

[4] Jones, C. C. (1995). A mechanical resonance apparatus for undergraduate laboratories. *American Journal of Physics,63*, 232-236.