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Nonlinear Harmonic Modes of Steel Strings on an Electric Guitar

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Nonlinear Harmonic Modes of Steel Strings on an Electric Guitar

Joel Wenrich

A THESIS

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Linfield College
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In partial fulfillment of the requirements
for the Degree of
BACHELOR OF SCIENCE
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Abstract

Nonlinear Harmonic Modes of Steel Strings on an Electric Guitar

Steel strings used on electric and acoustic guitars are non-ideal oscillators that can produce imperfect intonation. According to theory, this intonation should be a function of the bending stiffness of the string, which is related to the dimensions of length and thickness of the string. To test this theory, solid steel strings of three different linear densities were analyzed using an oscilloscope and a Fast Fourier Transform function. We found that strings exhibited more drastic nonlinear harmonic behavior as their effective length was shortened and as linear density increased.

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Introduction

The guitar is a stringed instrument that can be used to produce a wide range of notes by effectively changing the length of the set of strings, each of a different linear density. The strings are stretched over a strong piece of wood with one side round and the other flat, called the neck and fretboard respectively, with strips of metal wire set into the fretboard, called frets. On an electric guitar, vibrations of the metal strings are sensed by pickup coils, amplified by a simple circuit, and finally outputted. Below is a diagram highlighting the basic parts of an electric guitar. (See Fig. 1)

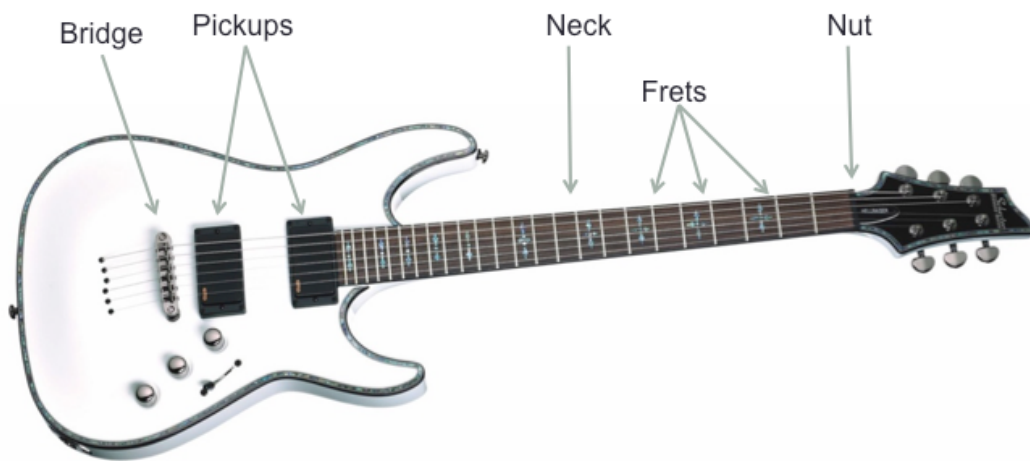


Fig. 1: Labeled diagram of an electric guitar^[15].

Like any musical instrument, guitars must be kept in tune so that they can be used to produce the quality of sound for which they were designed. Possible contributing factors to poor intonation have been studied such as dead spots^[5], tone decay^[10], and fret-string interactions^{[1] [13]}. Other factors include the harmonic

behavior of the vibrating strings [3][11][12][14]. In this paper, we will continue to build off of the work done with harmonics. Harmonics are particular frequencies that yield zero-amplitude points along a vibrating string. They are often analyzed as a standing wave (a snapshot of the wave at its greatest amplitude).

We study the harmonic vibrations of guitar strings and we believe there is a relationship between the nonlinear nature of harmonics on a string and its bending stiffness. This matters to guitar players and listeners alike because the presence of sharpening harmonic modes may become audibly sharp or flat when played in conjunction with other notes [7].

This is a study of steel strings. While most sets of strings are made of steel, other alloys such as bronze, copper, and nickel are used in the industry for the variety of tonal qualities they offer. The thicker strings responsible for producing lower frequency notes are not made from a single core of steel; rather, they consist of a softer metal – usually nickel or copper– wound around a steel core. Fig. 2 shows SEM images of the set of guitar strings used in this experiment:

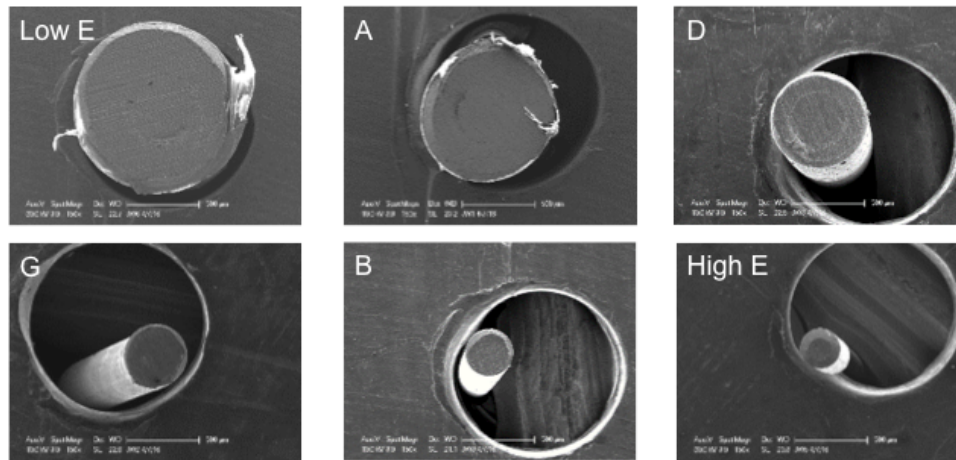


Fig. 2: SEM images of a standard set of guitar strings.

Using an electric guitar and a digital oscilloscope we test the harmonic behavior of different steel strings and compare it to theoretical results yielded through known relationships between harmonics and bending stiffness. Because wound strings (see Fig. 2: Low E, A, D) were not comprised of a single material, they were excluded from this study to avoid further complexity.

Theory

The Ideal String

To analyze the vibration of a steel string we must first understand the simplest case, an ideal string. An ideal string is infinitely thin (without bending stiffness) and has uniform linear density. When plucked, two wave trains propagate in opposite directions, reflecting off of the two fixed ends, which is the bridge and either the fret or the nut. These two wave trains continue to reflect back and forth, creating an interference called a standing wave. By the Principle of Superposition, the interfering waves are the sum

$$y(x, t) = \sin(kx - \omega t) + \sin(kx + \omega t) \quad (1)$$

Here, y is the wave amplitude, $\omega = 2\pi f$ is angular velocity, and $k = \frac{2\pi}{\lambda}$ is the wave number with λ being wavelength. By way of trigonometric identities, Eq. (1) can be simplified to

$$y(x, t) = 2\sin(kx) \cos(\omega t) \quad (2)$$

If we define the fixed ends to be at $x = 0$ and $x = L$, when a string is plucked, the only waves that continue any longer than a fraction of a second are the ones which yield an amplitude of zero at those positions. Note that in Eq. (2) the first term describes amplitude and the second term describes frequency. Here, it is important to note that the result of Eq. (2) is zero at both fixed ends, say $x = 0, x = L$. What

arises is an infinite amount of particular frequencies that yield positions along the string, x , that experience an amplitude of zero at all times; these points are called nodes. The n^{th} harmonic, denoted f_n , is a frequency that produces n number of nodes. Often in discussion of this topic, the term “harmonic” is synonymous with “mode.” The first three harmonics for an ideal string fixed at both ends are shown in the figure below. Also in the figure is the sum of the first three harmonics where it is important to note that the amplitudes are descending from the first to third harmonic. (See Fig. 3)

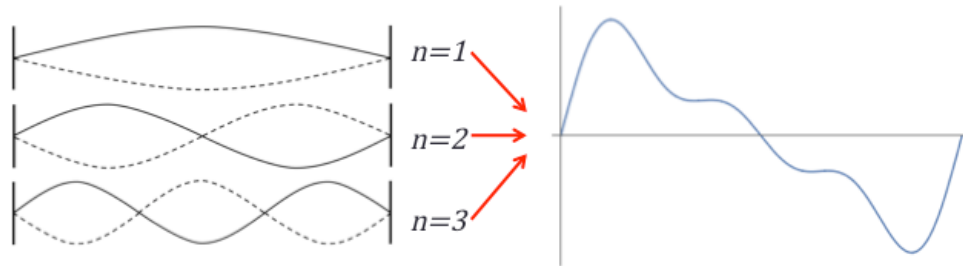


Figure 3: Standing waves of the first, second, and third harmonics (left); sum of the first three harmonics for one full wavelength plotted on time vs. amplitude (right).

Motivated to generate a relationship between harmonics and their corresponding modes, recall the speed of a transverse wave traveling on a string^[4],

$$v = \sqrt{\frac{T}{\mu}} \quad (3)$$

where v is wave speed, T is tension force, and μ is the string’s linear density. For such a string stretched between two end supports, natural modes of vibration are given by

$$\lambda_n = \frac{2L}{n} \quad n = 1, 2, 3 \dots \quad (4)$$

where λ_n is wavelength of mode n with string length L . From figure 3 and Eq. (4) a critical concept arises: all harmonics occur when a string is initially plucked. Again, there are infinitely many frequencies that appear when a string is first plucked but most of them are damped by the bridge–fret/nut interaction. Speed and wavelength are both related to frequency, f , through

$$f_n = \frac{v}{\lambda_n} \quad n = 1, 2, 3 \dots \quad (5)$$

By substitution of Eqs. (3) and (4) into (5),

$$f_n = \frac{n}{2L} \sqrt{\frac{T}{\mu}} = n f_0 \quad n = 1, 2, 3 \dots \quad (6)$$

Here we have a way of calculating the frequency of each mode of vibration for a string with specific properties. We refer to f_0 as the fundamental frequency. Notice the positive linear relationship between frequency and mode. In the ideal case length, tension, and linear density are assumed to be constant.

Bending Stiffness

The ideal string model breaks down when we no longer make the assumption that the string is infinitely thin. As a result, harmonics do not add together to produce a wave that is consistent. Complication ensues when we incorporate thickness and material properties of the string^[4]:

(7)

$$f_n = nf_0\sqrt{1 + Bn^2} \left[1 + \frac{2}{\pi}\sqrt{B} + \frac{4}{\pi}B \right]$$

with

$$B = ESK^2/TL^2 \quad (8)$$

being bending stiffness. Here, bending stiffness is a property of a string of given material with E , modulus of elasticity, S , cross-sectional area and radius of gyration, which for a cylinder of radius a is $K = a/2$. Clearly, Eq. (7) is a departure from the linear relationship between frequency and mode. It is this nonlinear equation that we will use later to compare with results obtained from our experiment.

Music Theory

We care about the sharpening of harmonics because there are potential ramifications for the purity of notes as they are played together. Consider two separate strings of length L fixed at both ends. If we take one string and stop it halfway so that its new length is $1/2 L$, we would be able to produce a note that is twice the frequency of the note produced by a string of length L . This note is called an ‘octave.’ Notice that the frequency of the octave is the first harmonic of the original note. If we were to take a string of the same length and stop it at $2/3 L$ we would produce a note that is called a ‘fifth.’

Octaves are notes separated by an interval of twelve half steps. The interval of perfect fifths, as it’s called, consists of notes separated by seven half steps. Using this interval, chords can be made by playing two or more of these fifths together. Hence, it is crucial that chords are comprised of notes that are in tune or they will

suffer a beat frequency, which is the periodic oscillation in total amplitude of a compound wave comprised of multiple component frequencies ^[8].

Experimental Methods

In this experiment, the measurement procedure consisted of plucking a guitar string and obtaining the component frequencies of the signal using the Tektronix TDS 2002B oscilloscope. Figure 4 shows a schematic of the experimental setup.

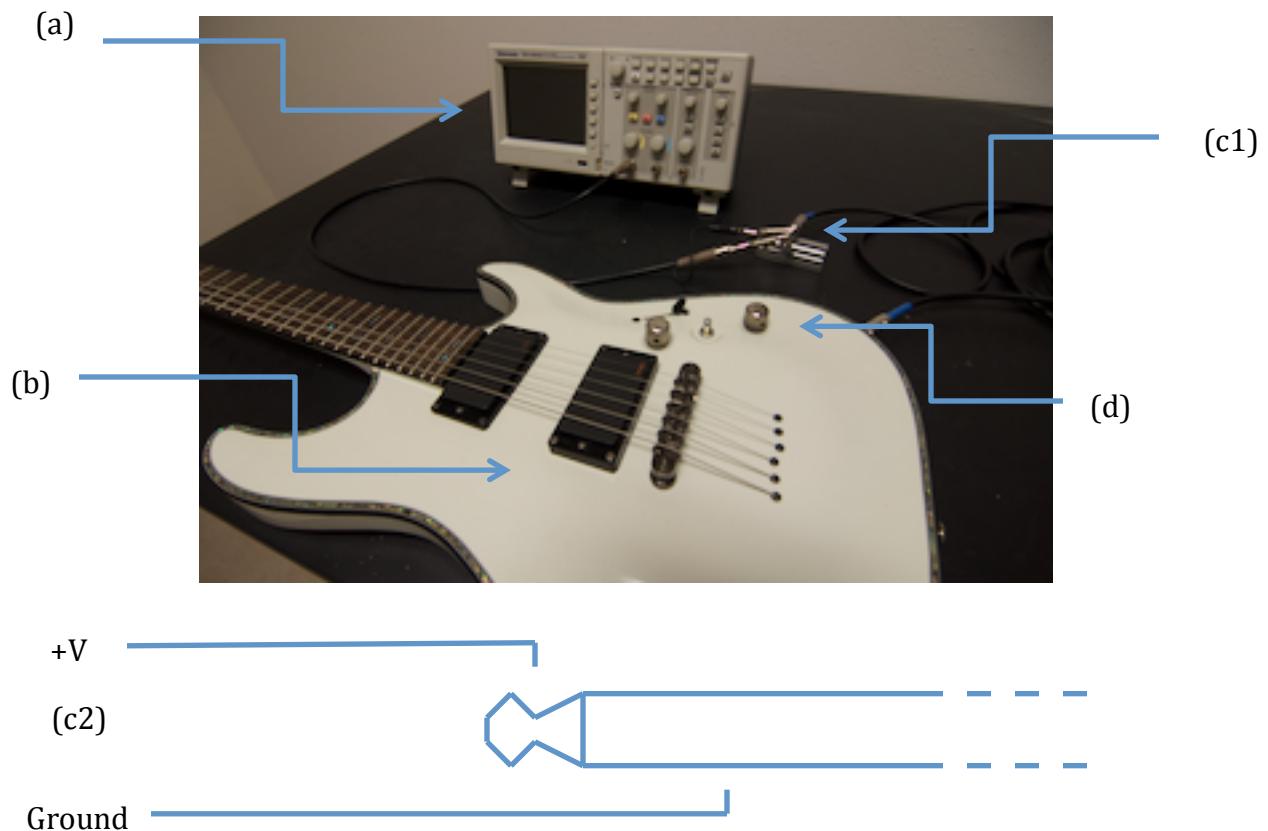


Fig. 4 Schematic of experimental setup. (a) Tektronix TDS 2002B Oscilloscope. (b) Schecter C-1 Electric Guitar. (c1) Connection between guitar cable and Channel 1. (c2) Expanded diagram of connection (d) Connection between guitar and guitar cable.

The Tektronix TDS2002B Oscilloscope served the purpose of this experiment well because of its Fast Fourier Transform (FFT) and USB functions. Three parameters of the FFT are adjustable: source, window and zoom. Source

determines which channel is sampled. Only one channel was used in this experiment. Three types of window settings determined frequency resolution and amplitude accuracy. Both measuring periodic waveforms, the “Hanning” and “flattop” window modes most accurately judged frequency and amplitude, respectively. The third window option was “rectangular”, lending special attention to waveforms without discontinuities. Catering to the focus of the experiment, “Hanning” was the optimal choice. Zoom determines horizontal magnification of the data with options of X1, X2, X5, X10. Once set, window and zoom were held constant whereas sampling rate was cycled through when collecting a set of data. The oscilloscope’s range was between 5.0 samples per second (S/s) and 2.0 GS/s but for the purposes of measuring frequencies produced by a guitar a range of 1.0 kS/s to 50.0 kS/s was sufficient. Each sampling rate scanned 0 Hz. to the Nyquist frequency, which is half the sampling rate, to avoid aliasing. More detailed information can be found in the oscilloscope’s manual.

With the oscilloscope tuned, special attention had to be paid to the guitar and how each string was plucked. A standard 1.0mm guitar pick was used to excite the strings during the experiment. As Politzer suggests ^[11], the angle at which a string is plucked relative to the fretboard affects the intensity of vibration immediately after it is plucked. It turns out that plucks parallel to the fretboard produce the highest intensity vibrations with the least decay. Thus, in order to stay consistent with the plucking hand as best as possible without some sort of automated process, the experimenter did entire sets of data at a time being careful to maintain steady hand position, plucking force and plucking position along the string.

Determining the moment at which the sample was taken required some technique as well. Immediately after a string is plucked there is a “twang,” as it is referred to, that is the audibly positive shift in frequency due to what Errede suggests is the extreme initial amplitude generated by the pluck [3]. Because of this effect, which lasts less than a tenth of a second, the proper sample to take was just after the pluck. This entailed setting the oscilloscope automatic trigger and pressing STOP on the second or third sampling depending on the rate. For the slower rates, such as a 1 kS/s and 2.5 kS/s, the sampling time was too long to take the second frame because by then the intensity of the vibrating string decayed beyond measurement.

With the oscilloscope and guitar prepared for data, samples could then be taken. As previously described, the sampling of the oscilloscope was paused just after the initial pluck of a string. Utilizing the USB function of the oscilloscope, the waveform data was saved onto a flash drive. Organization was critical due to the volumes of data collected. Data was separated first into folders differentiated by string then into folders according to fret. Data was not separated into their own subfolders based on sampling rate because such information was included in the waveform file.

The oscilloscope saved the data as a Comma Separated Document (CSD) where each point was a coordinate of frequency and intensity. These data were then put into the computer program, Origin, in order to better fit the points. Excel was used to keep a list of the fitted points. To bring it all together, Mathematica

proved useful in calculating theoretical data, which was compared directly to the experimental data. Greater detail of the analysis is described in the next section.

Results & Analysis

In this experiment only the three thinnest strings on the guitar (G, B, High E) were analyzed because they were solid steel whereas the other three strings (Low E, A, D) have copper or nickel winding around solid steel cores. The physical properties of the strings used are provided in Table 1 below.

Table 1 Physical properties of G, B, and High E strings used in the experiment. Diameter was obtained using veneer calipers, Tension through manufacturer data, and f_0 from the oscilloscope.

	String		
	G	B	High E
Diameter (mm)	0.406	0.279	0.228
Estimated Tension* (N)	65.3	49.0	58.4
Measured f_0 (Hz.)	194.8	246.2	327.6

**Tension while in standard tuning and on fret zero, as per the manufacturer of the strings used,*

D'Addario XL120.

Each set of data began as a reading from the oscilloscope in the form of 1024 points spanning a tunable range of frequency. Sampling rates used in this experiment varied from 1.0 kS/s to 50.0 kS/s with respective intervals between data of 0.488 Hz. to 24.41 Hz. Figure 5 is an example of raw data collected from the oscilloscope. Note that although there are spikes in the data they are not reliable due to the sampling interval. Generally, the peaks close to the maximum threshold were ignored because data from faster sampling rates suggested the actual peak was slightly beyond the threshold.

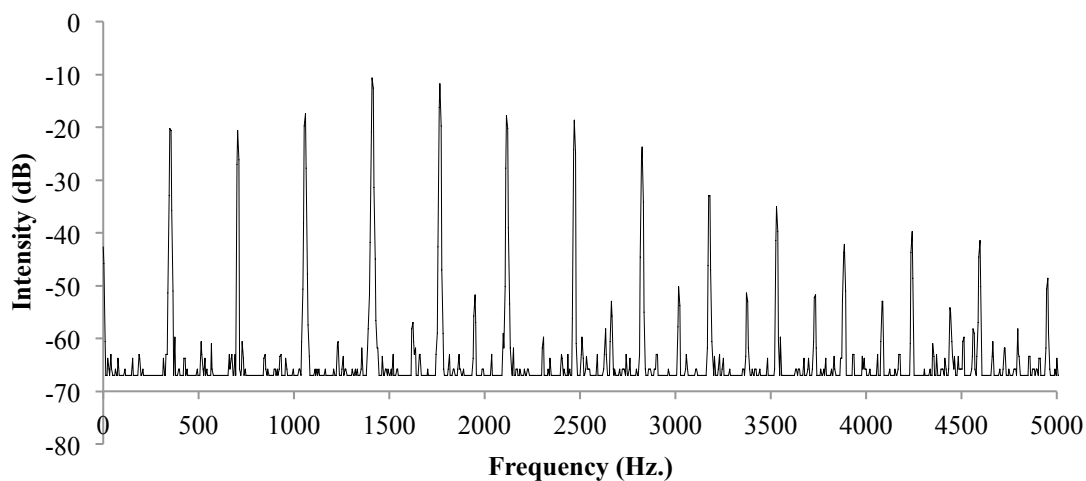
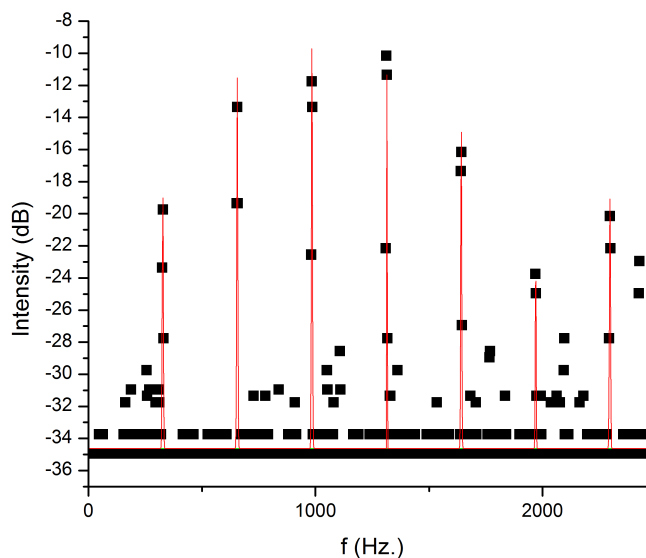


Fig. 5 Raw data collected from the oscilloscope. High E String, fret 1, 10.0 kS/s

Better accuracy could be obtained from the raw data by using Origin to fit Gaussian curves to each peak. Using the “Fit Multiple Peaks” function, up to 12 peaks at a time could be modeled. For some of the peaks that were of low intensity but higher than that of noise, the function “Fit Single Peak” was required. Figure 6 shows a snapshot of both a multiple peak analysis and single peak analysis.

Fig. 6 Experimental data fitted using Origin. High E String, fret 1, 10.0 kS/s



After each fit of the raw data, the values of the peaks were recorded in Excel spreadsheets. It was these data that we considered to be our experimental data.

The following figures exhibit experimental data and accompanying theoretical curves calculated using Eq. (7) along with the information from Table 1. For each string, results for frets 0, 12, and 24 are given. The three frets were chosen because they correspond to the first three harmonics— f_1 at the full length, f_2 at half-length, and f_3 at one-third length. The x-axis is the mode, n , and the y-axis is frequency of mode, f_n , divided by the mode. The motivation for presenting the data in such a fashion is that it makes more visible an increasing shift in harmonics, which is what we would expect to see in a string with high bending stiffness. Note that an ideal string would be a horizontal line with the only y -value being the fundamental frequency. By Eq. (7) we would expect to see the most drastic positive shifts for fret 24, then fret 12, then fret 0, regardless of string. Additionally, the same equation suggests that there would be greater shifts in the thicker strings on any given fret. Note that in the discussion of these figures any shift in frequency describes a deviation from the fundamental frequency when divided by the corresponding harmonic number, n . Therefore, each point represents a frequency shifted n times away from what the frequency would ideally be at mode n . Graphs can be compared in this manner because the x-scale is the same in every graph for the Figs. 7, 8 and 9 below.

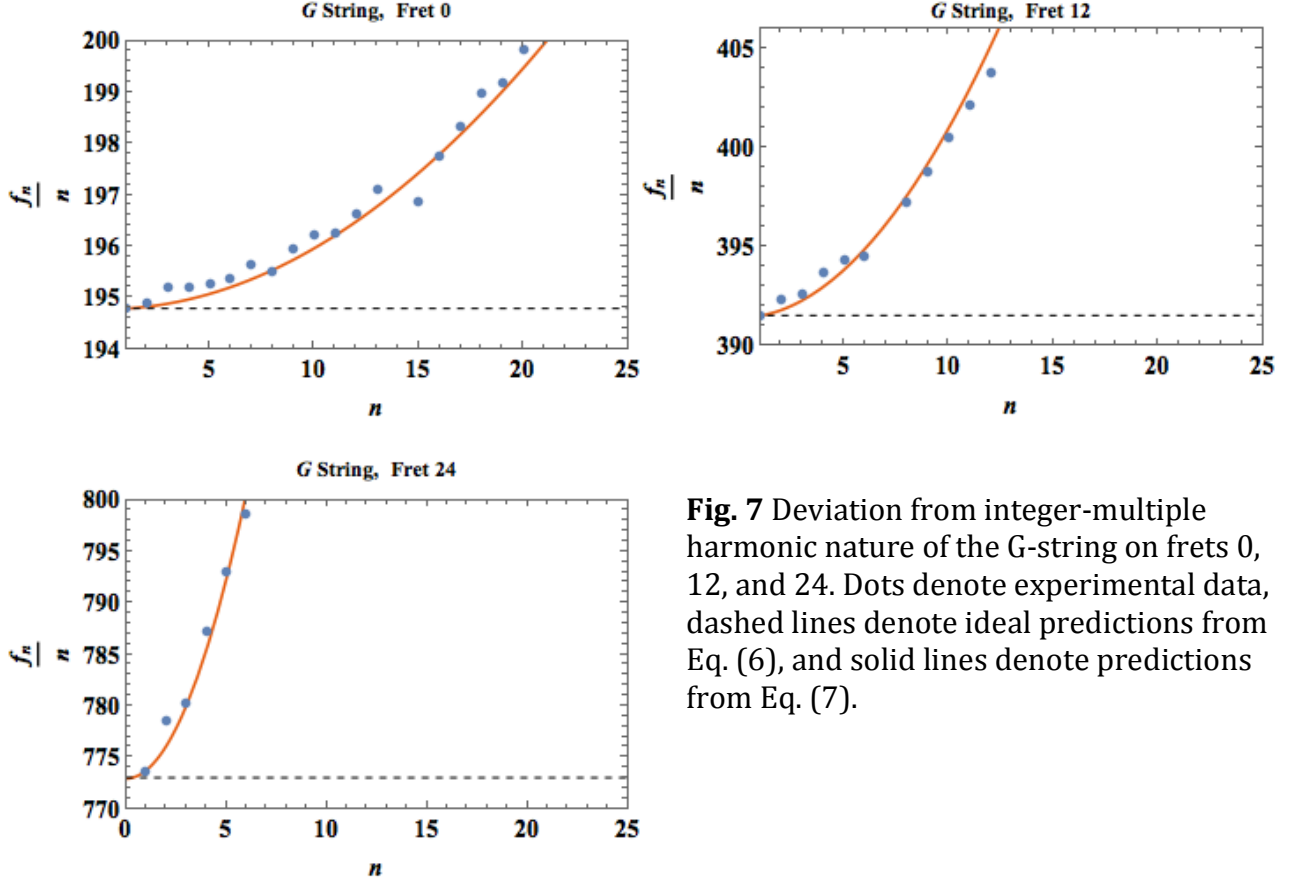


Fig. 7 Deviation from integer-multiple harmonic nature of the G-string on frets 0, 12, and 24. Dots denote experimental data, dashed lines denote ideal predictions from Eq. (6), and solid lines denote predictions from Eq. (7).

From Fig. 7 we can see that as the string length decreases (fret number increases) the harmonics shift more significantly. On the 0th fret we see a +5 Hz. shift at $n = 20$. For the 12th and 24th frets we see a +5 Hz. shift reached at roughly $n = 8$ and $n = 3$ respectively.

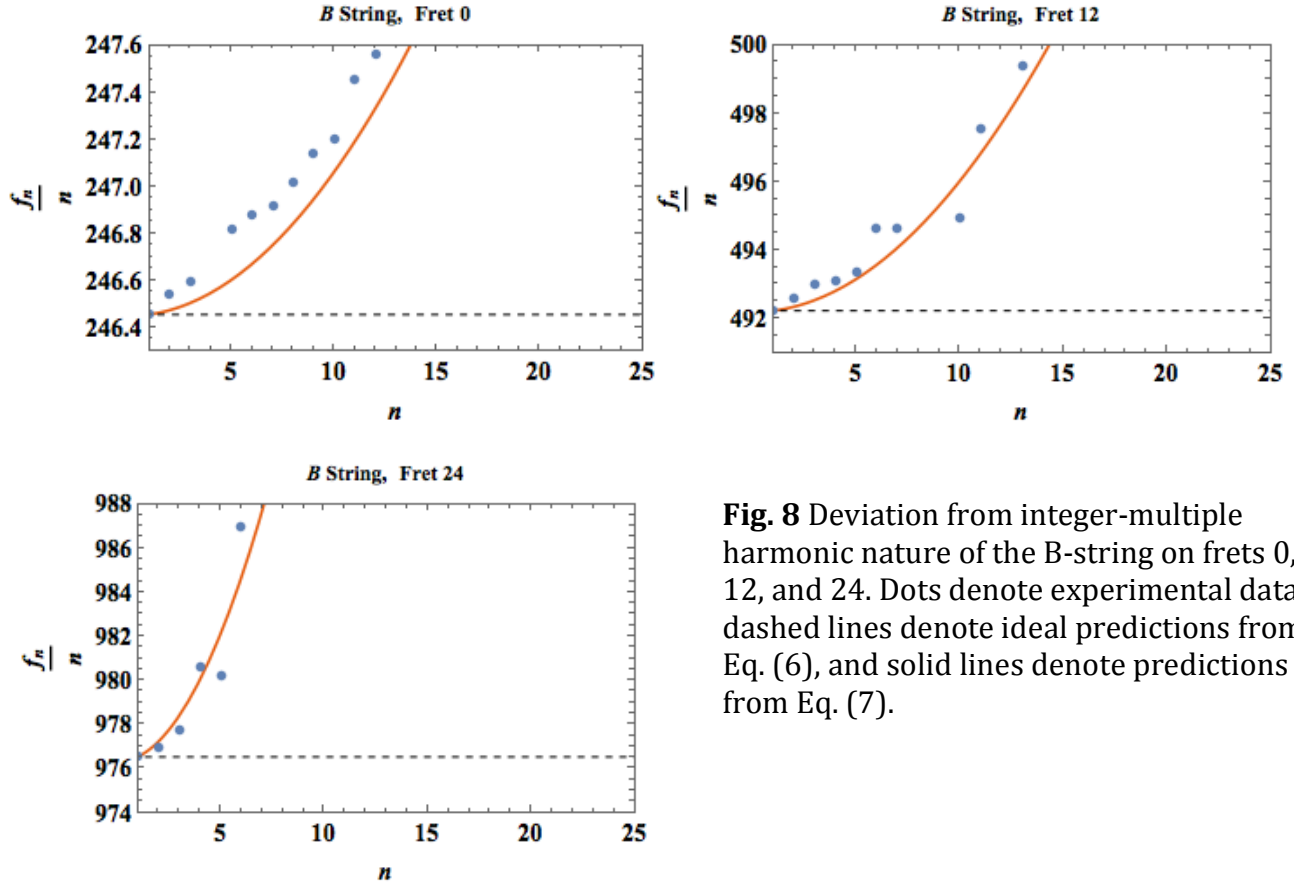


Fig. 8 Deviation from integer-multiple harmonic nature of the B-string on frets 0, 12, and 24. Dots denote experimental data, dashed lines denote ideal predictions from Eq. (6), and solid lines denote predictions from Eq. (7).

Figure 8 continues the trend observed in Fig. 7 but for the B-string. Comparing the two, we see that in each case between the 0th, 12th and 24th frets, the thicker G-string displayed a more drastic shift.

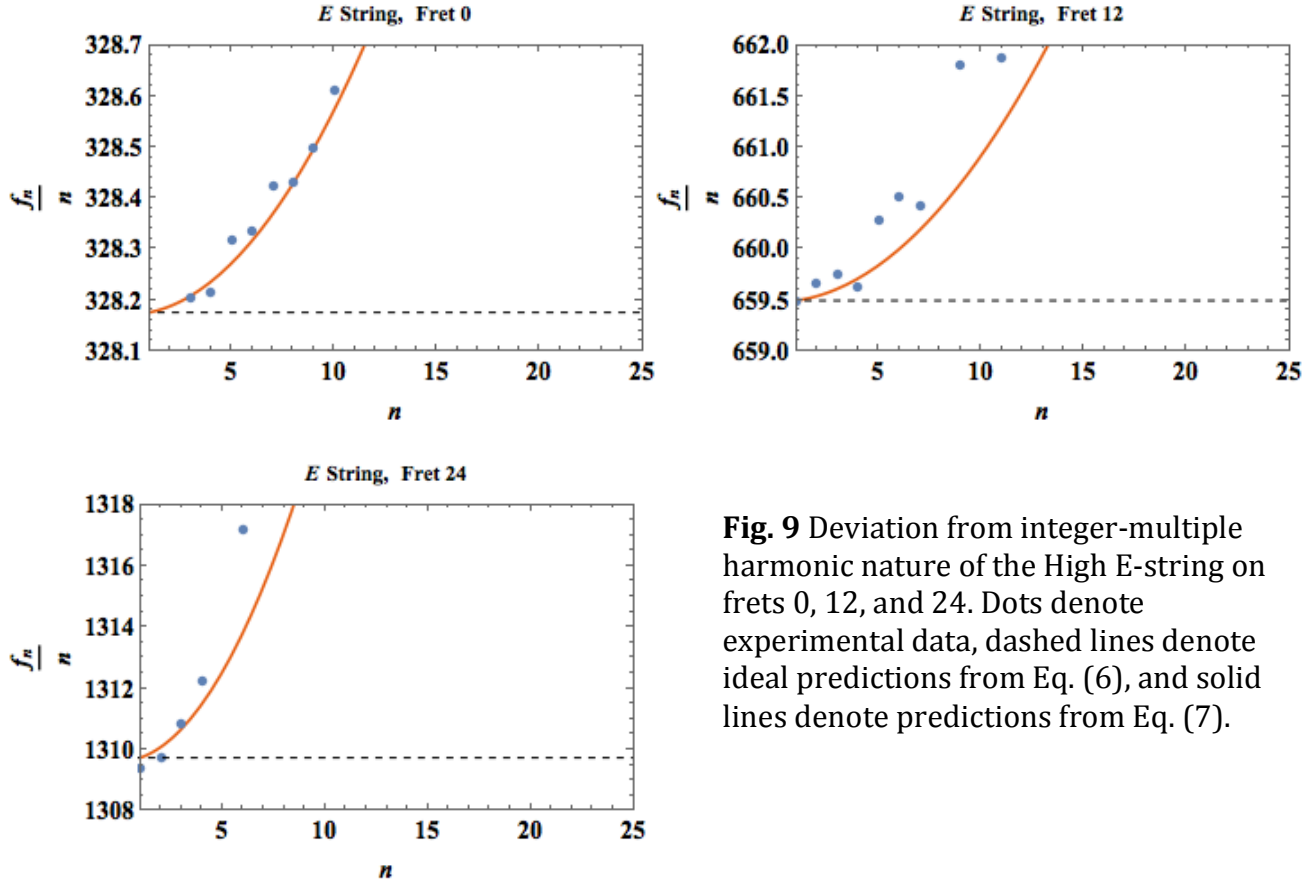


Fig. 9 Deviation from integer-multiple harmonic nature of the High E-string on frets 0, 12, and 24. Dots denote experimental data, dashed lines denote ideal predictions from Eq. (6), and solid lines denote predictions from Eq. (7).

With the third and final set of data, Fig. 9 completes the trend observed in the previous two figures. As the thinnest of the strings, the High E-string exhibits the least drastic of the shifts for each fret. From these figures we observe proportionality between nonlinear harmonic behavior and both the inverse of length squared and cross sectional area as suggested by Eq. (7). To highlight the inverse of length squared dependency we can look at each string as an individual case and see that as the length of the string decreases we see a greater positive shift. As for the cross sectional area dependency, we can look at each fret as an individual case and compare the harmonic shifts for each string. Indeed, we see that for each fret the G-string had the most significant shift,

followed by the B-string, and lastly by the High E-string (descending order of thickness).

When both contributing factors are taken into account, direct comparisons between strings can be made for a given note. In Fig. 10, the same note (High E) was played on each of the three strings. Both length and linear density are being varied in this example.

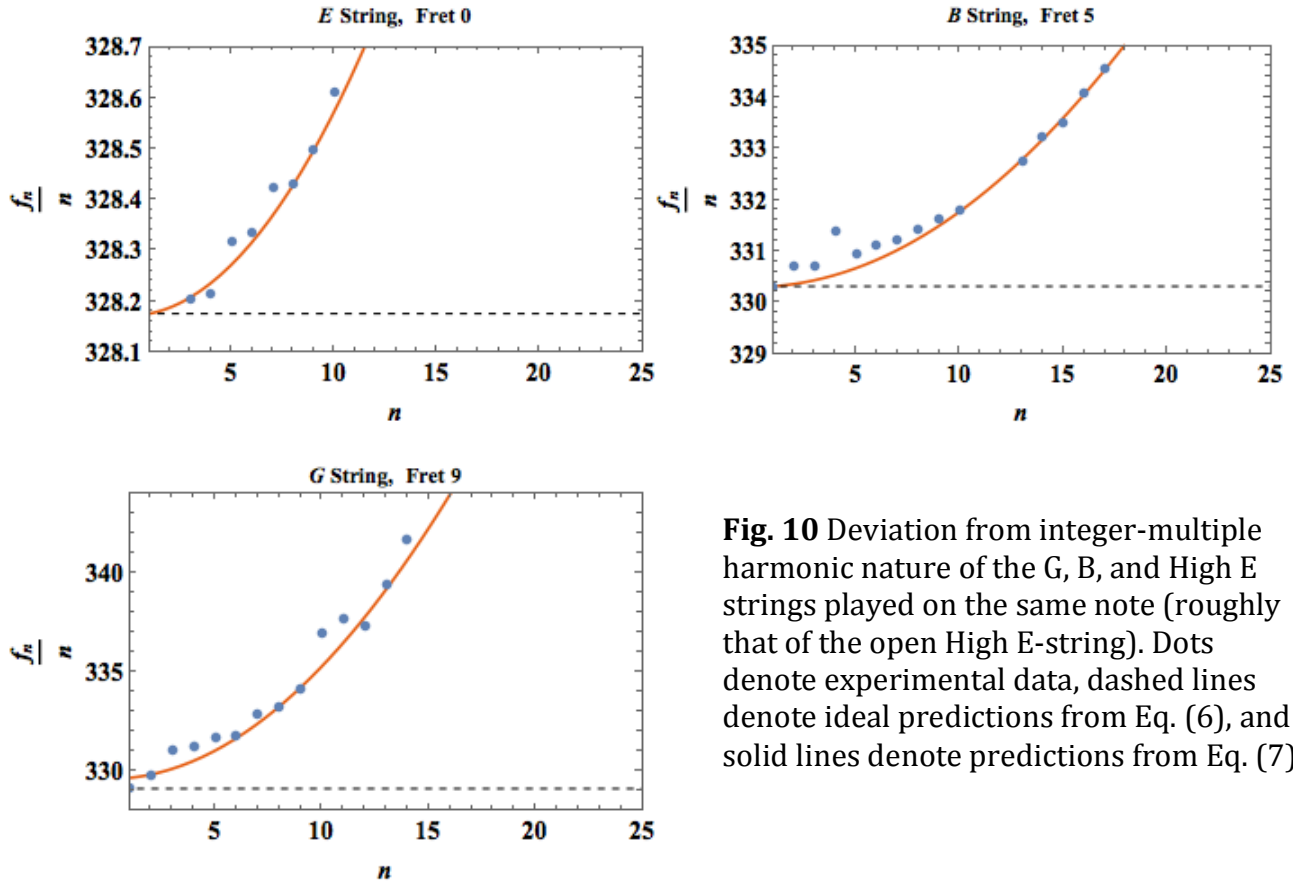


Fig. 10 Deviation from integer-multiple harmonic nature of the G, B, and High E strings played on the same note (roughly that of the open High E-string). Dots denote experimental data, dashed lines denote ideal predictions from Eq. (6), and solid lines denote predictions from Eq. (7).

Figure 10 concurs with the behaviors observed in the previous three figures. These results suggest that one can optimize the intonation of their play, given the option to play the same note on different strings.

Error in the data stems from the potential lack in low-frequency sensitivity of the oscilloscope as well as the possibility that the pickups^[6] and circuitry of the electric

guitar are not built to be sensitive to frequencies far above the third harmonic of the High E string (roughly 1318 Hz.).

Conclusion

Nonlinear harmonic behavior was observed in the G, B, and High E strings of an electric guitar. In accordance with the prediction of Eq. (7), results show that the degree of nonlinear harmonic behavior is greater as string thickness increases and length decreases. Our observations agree with that of Politzer ^[11]. From the results, it is possible that observed deviations in harmonics would be perceivable (see ^[9]). Further research could include testing wound strings for similar behavior.

Sources of error in this study vary from human measurement to instrumental limitations. It was not possible to maintain perfect consistency in plucking the strings so there may have been harmonics that were too faint to observe. Better equipment could have been used because our oscilloscope was only able to achieve a precision of ± 24.41 Hz. at 50.0 kS/s. This sort of error was only significant in low-frequency measurement, which was critical because the theoretical lines in Figs. 7, 8, and 9 were calculated based on the fundamental frequency. Ideally, we would be able to scan that range of data with a precision of closer to ± 0.5 Hz. Because we chose to analyze the vibration of the strings through electronic means (guitar and pickup system), it may just be that electric guitars are not made to register harmonics far beyond the fifth or sixth. It would be interesting to replicate the experiment with an acoustic analysis instead and compare results.

It is not tangible to eliminate steel as the main metal used in guitar strings but out of studies such as this, there may develop new techniques for compensating this effect. Perhaps strings of non-uniform density could be designed specifically to minimize skew in harmonics all throughout the length of the fretboard.

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