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Expanded Parameters in the Self-Organized Critical Forest Fire Model

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Expanded Parameters in the Self-Organized Critical Forest Fire Model

Riley Self

A THESIS

Presented to the Department of Physics
LINFIELD COLLEGE
McMinnville, Oregon

In partial fulfillment of the requirements
for the Degree of

BACHELOR OF SCIENCE

May, 2016

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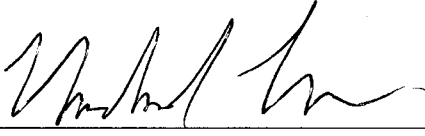
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ABSTRACT

Expanded Parameters in the Self-Organized Critical Forest Fire Model

The forest fire model has been used to test the theory of Self-Organized Criticality as a model of complexity. The goal is to search for scale invariance in randomly generated forest fires using a computer simulation. In a previous model by B. Drossel and F. Schwabl [1], power-law behavior was seen when the nearest neighbors to a tree on fire catch on fire, and it has been assumed that if further trees also catch fire, then it will still exhibit self-organized criticality, showing scale invariance. Testing this assumption aids to the exploration of the applicability of self-organized criticality because the model is the most useful when it applies to a large range of systems, as closely related to nature as possible.

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Chapter 1

Introduction

1.1 Self-Organized Criticality

Self-Organized Criticality (SOC), first conceptualized by Per Bak [2], is a way of characterizing and finding order in random systems. It is explored by creating different models with a critical point, where the system suddenly is not in equilibrium, that abide by different sets of rules. Some models have included avalanches, traffic jams, earth quakes, and in this case, forest fires [3]. By looking at different variables in these systems as they reach a critical state, these systems actually show distinct trends. These trends are called power-law behavior and scale invariance.

Power law behavior is when one quantity varies as a power of another. When a system is expanded, the power-law relationship should be the same, meaning that the relationships should vary with the same power no matter the system size. In order to search for SOC, one must test for both the power-law relationship and scale invariance [4].

1.2 Forest Fire Model

First modeled by B. Drossel and F. Schwabl [1], the forest fire model is used as a way of exploring the potential applicability of SOC. Rather than being an accurate model of a forest fire, it is a simple representation of a forest fire that is used as a model for showing how large scale structure can arise in dynamical systems [5]. In this model, fire hits a random tree on a lattice, and the fire spreads to the nearest neighbors from there. In the end, we look for SOC in the number of clusters of trees unaffected by the fire and the size of these clusters. In the Drossel-Schwabl model, both scale invariance and power law behavior was found. They stated that if the fire were to spread further trees than in there model, then the power-law relationship would exhibit the same trend line. This paper tests this claim.

By testing for Self-Organized Criticality under different system rules, it becomes a more unifying theory, and comes closer to being useful when looking at more accurate systems in nature [6].

Chapter 2

Self Organized Criticality as a Physical Model

Self-organized criticality has evolved as a way to explain complex systems in nature by looking for power-law relationships, where one variable varies as a power of another, are found when looking at the system overall even though small scale interactions are unpredictable. It has been explored in systems from protein folding to earthquakes. It is a unifying theory used to relate a wide range of systems.

2.1 Sand Pile Model

The classic example used to describe self-organized criticality is the sand pile model [2]. In this model, we add sand grain by grain, gradually forming a pile. After a while, a certain grain will cause an unstable state by falling, knocking over other grains, creating an avalanche. This then results in a rearranged system, sometimes only slightly from small avalanches, but sometimes drastically from large avalanches. At some point, the pile will reach a critical state where larger avalanches occur that span the whole pile. This is described as the self organized-critical state. A critical state is the point in which a system is no longer in equilibrium. Self-organization is

when a larger system shows order due to smaller scale interactions. The sand pile is critical because small random landslides show order when looking at the system at large, and it is self-organized because the avalanches occur spontaneously. Thus, the pile has changed from a stable system to a critical system.

2.1.1 Mathematical Application

Theoretical physicists have found the sandpile model mathematically interesting because the distribution of the size of the avalanche and the frequency of the avalanche display a power-law behavior. As said before, power-law behavior happens when one variable is a power of the other variable. The power-law behavior was found when looking at the avalanche size s , and the frequency f [2].

$$S \propto \frac{1}{f^\alpha} \tag{2.1}$$

This basically means that the smaller avalanches are much more frequent than the bigger avalanches, and the frequency of this follows a power-law behavior. This is interesting because the relationship remains, regardless of time scales and other variances without adjusting any parameters.

It is important to note that when the size of the system in a simulation changes, the power law behavior does not. When this is represented on a log-log scale, then the slope of the line should be the power of the equation, across all system sizes, which is defined as scale invariance. In this example, the α value should not change if there are more grains of sand added to the pile.

2.2 Application to the Forest Fire Model

Since the original sandpile model, self-organized criticality has been applied to many different systems in nature as potentially a unifying theory, connecting the behavior

of many types of systems. In this paper, we will be using the general rules of a forest fire to see if this model exhibits self-organized criticality. In the forest fire model, a random tree is caught on fire, spreading the fire to neighboring trees, and the fire spreads on from those trees. Over time, other trees are randomly caught on fire due to lightning striking based on the probability of there being a spark, which is a set parameter value.

Throughout the burning, clusters of trees that have not been caught on fire are established. In order to analyze the Self-Organized Criticality of this system, we will count the number of clusters at any given point in time, as well as the radius of the cluster as a function of the cluster size. The number of clusters is how many groups of unburned trees there are, the cluster size is how many trees there are in a cluster, and the cluster radius is the radius if the cluster is assumed to be a circle.

The cluster radius as a function of cluster size should be invariant no matter what because radius is a variable of size when looking at a circle, so we should expect the same slope on a logarithmic scale. It is useful as a visual and a comparison to the number of clusters as a function of cluster size because that relationship is not so obvious. We would expect there to be relatively fewer clusters when the clusters are larger because they span a larger portion of the forest, but there is no exact equation that provides a clean relationship like the cluster radius does.

If the system follows the expected Self-Organized Criticality model, then the power law relationship and scale invariance should appear when looking at the number of clusters and cluster radius as functions of cluster size [7]. Here, we are testing for scale invariance with two models with one slight change in the rule, modeled in the following figure.

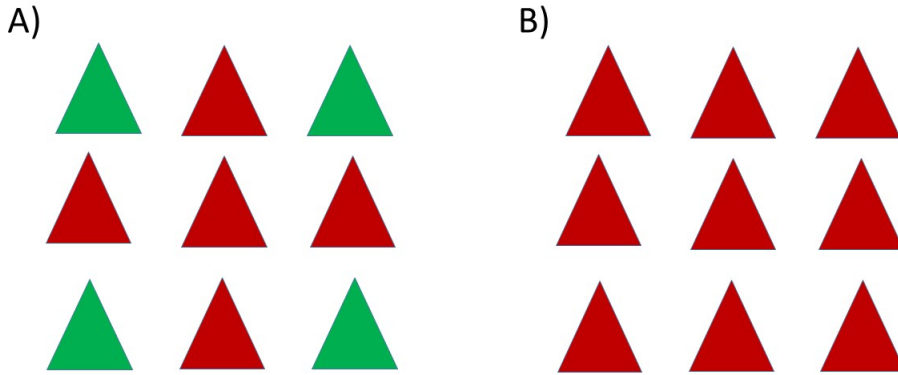


Figure 2.1: Model comparison with the original Drossel-Schwabl model (a) and the expanded parameters (b).

Drossel and Schwabl [1] write that the forest-fire model being critical is not dependent on parameter values, and should still hold the power-law behavior with modifications of the rules such as another lattice symmetry or a fire spreading to the next nearest neighbors. They found power law behavior and scale invariance when looking at the number of clusters N as a function of cluster size s .

The goal of this paper is to test these claims by expanding the parameter values. In the Drossel-Schwabl forest fire model, the fires only spread directly to the trees that are left, right, up, and down from the tree on fire. To expand the parameters, we have adjusted the model to also spread to the diagonal trees. If the same power-law behavior appears, along with scale invariance, then we have further supported the claim that the forest fire model is a self-organized critical system. As more parameter values are added, we come closer to applying Self-Organized Criticality to a system in nature, rather than just models. At this point, the forest model is more of a metaphor, where we are looking at a system with different rules from other models where self-organized criticality was found in the past.

Chapter 3

Methods

In order to create this forest fire model, an array of numbers is made to represent a grid of trees. For simplicity in the explanation, we will use a 10x10 set of numbers, creating 100 locations. Each location is randomly assigned a 1 or a 0. If there is a 1, then there is a tree in that position. If there is a 0, then there is not a tree.

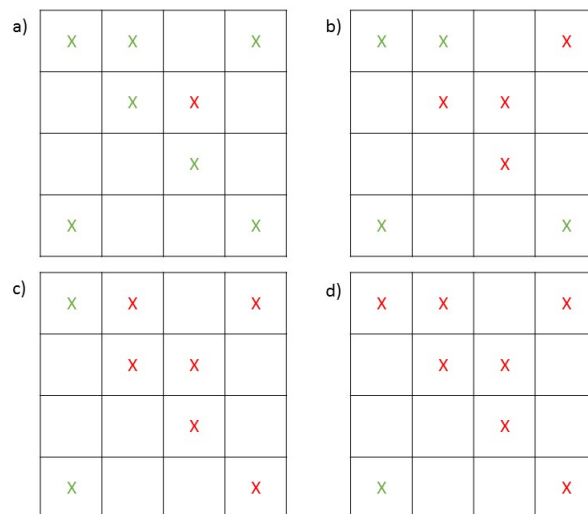


Figure 3.1: Four time-steps in a randomly generated forest fire with (a) random tree being set on fire (b-d) the fire being spread to the nearest neighbors, leaving one tree left.

The occupation probability in the code determines the probability that there is a 1 in any given location, controlling the forest density. If the occupation probability is 0.6, then there is a 60% chance that there is a tree in a single location, meaning that the forest is roughly 60% filled with trees. In this model, we set the occupation probability to be 40% to be consistent with previous models. This changes how fast the fire spreads throughout the forest, which should not change the power-law relationship because theoretically, expanded parameters should not change the outcome of SOC. After that, the fire needs to be started on a single tree. This is done by randomly placing a single 2 in the array in place of a 1, representing a tree on fire. In each time step, all 1's surrounding a 2 will then catch on fire. This is demonstrated in Figure 3.1 (b).

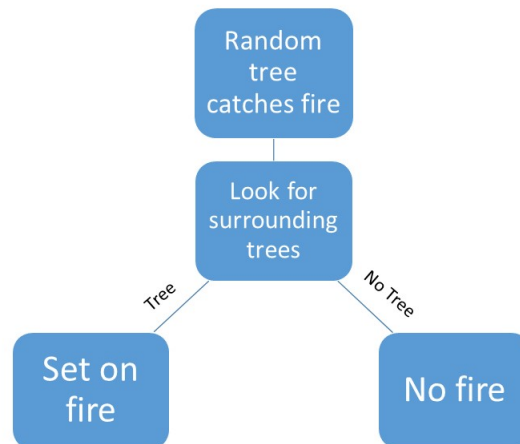


Figure 3.2: Loop in code, repeatedly looking for surrounding trees and setting nearest neighbors on fire.

In the next time step, all of the 1's around the new 2's will catch on fire. This is shown in figure 3.2. The occupation probability decides the density of the forest, and can then be adjusted to change how much the fire spreads. The fire will keep spreading until there is nowhere else for it to spread to. If there is nowhere for the fire

to go, that means that there are no 1's surrounding any 2's, or the fire has completely spread to the whole system. This is a closed system, so the boundaries end at the edge of the lattice.

In addition, a lightning parameter was added, meaning that a random tree could catch on fire based on a certain probability at any time throughout the simulation. This feature is necessary when analyzing forests with a small occupation probability so that fire will keep spreading when one chain stops.

From here, the clusters are explored. At various time-steps throughout each run, the number of trees in each cluster of non-burned trees is counted, as well as the total number of clusters in the run. A cluster is defined as a group of trees next to each other without a burnt tree in the way. A cluster can be as small as one tree and as large as the whole forest. Ideally, the code would be programmed to count the clusters and track cluster sizes. This code was started by finding all of the 1's in the array, and creating a new array of 1's in that location. Then, it looks at the first element in that array, and checks to see if its nearest neighbors are also in the array. The problem with this code was that it could only look at the next nearest neighbors in the array, and would not expand to the whole cluster. This problem would create too much error in the counting, so the data was hand collected instead. This was done by hand counting the number of clusters and the number of trees in each cluster over incremented time-steps over many runs through the code. Then, the radius of each of the clusters is calculated by using the number of trees as the area, and using the equation of the area of a circle to find the radius. We will plot the mean number of clusters $N(s)$ and mean cluster radius $R(s)$ as functions of cluster size s , expecting a power-law scale.

Chapter 4

Results and Analysis

Each run through the code created a forest fire such as the one in figure 4.1.

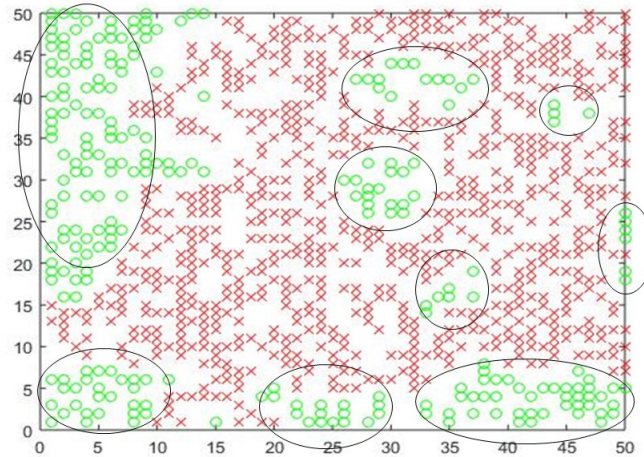


Figure 4.1: Example of a single time step in the forest fire, with clusters circled.

The figure shows what is considered a cluster when collecting data. Figure 4.1 has 9 clusters of non burned trees. Each cluster is counted and then the average of those is taken to produce a single data point. Then, the radius is found by taking the square root of the total number of trees divided by π . This was done over many different time steps and runs through the code in order to achieve the results.

My results show that the power law appears for both the number of clusters and the cluster radius as a function of the cluster size, but it does not support scale invariance due to the different slopes. This does not support the idea that Self-Organized Criticality is seen in forest fires when the parameters are expanded with the fire spreading to further trees.

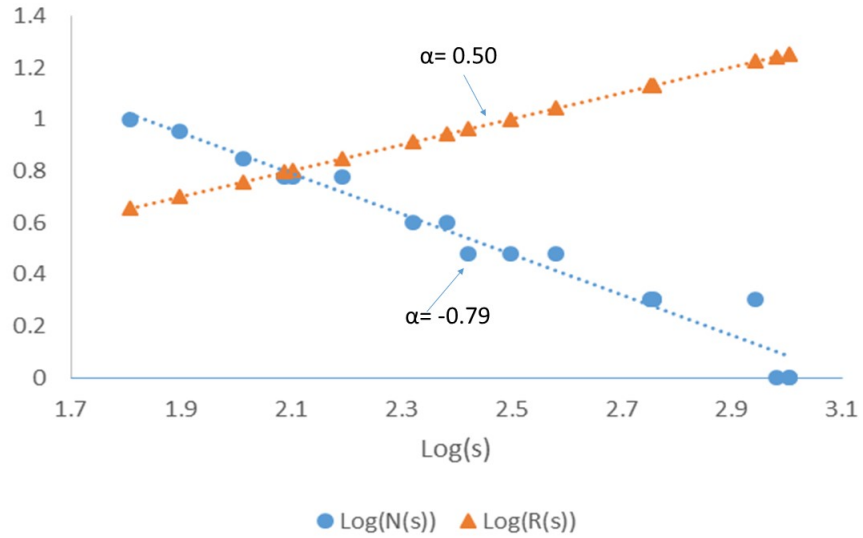


Figure 4.2: Mean number of clusters and mean cluster radius as functions of cluster size when the fire only spreads to four locations.

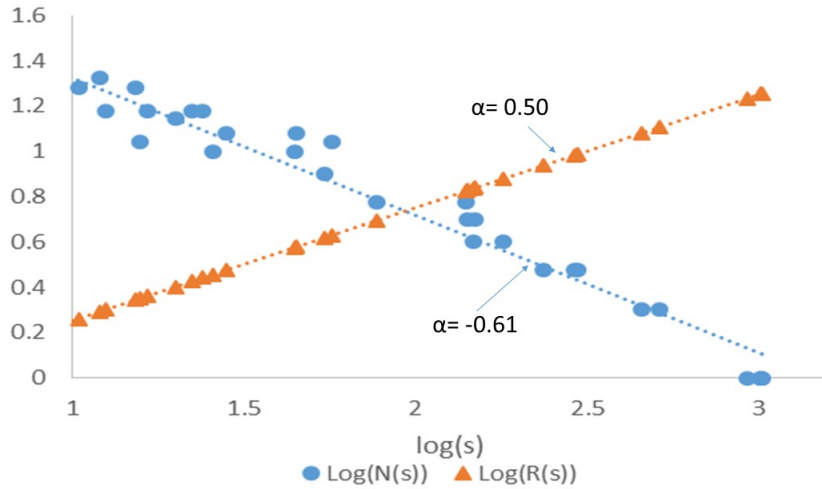


Figure 4.3: Mean number of clusters and mean cluster radius as functions of cluster size when the fire spreads to 8 locations.

In both Figure 4.2 and 4.3, the forest was a 50x50 grid, set with a 40% forest probability density, and 1.2% lightning probability. Figure 4.2 is an imitation of the Drossel-Schwabl model, while Figure 4.3 is the model with expanded parameters. Looking between these models, we see different slopes for the number of clusters as a function of cluster size, which means that we do not see scale invariance. We do still see power-law behavior with only a 4% error from the best fit line in both data sets. This error is not large enough for the slopes to potentially support scale invariance. The power-law the potential for SOC to still be exhibited when you change the size of the system, but leave the rest of the parameter values the same.

We do still see the power law behavior and scale invariance when looking at the radius of a cluster as a function of cluster size. this makes sense, because intuitively the radius varies with the cluster size since they are dependent on each other. representing this on the graph is still useful as a way of visualizing how the number of clusters as a function of cluster size varies.

These results point in the direction of showing that scale invariance does not

appear when looking at two models with different parameter values. This neither supports or denies SOC in the forest fire model because it is possible that when looking within a model with set parameters, scale invariance still appears when the size of the lattice is changed.

Discrepancies in the data could have appeared because the code was unable to actually count clusters, so they were hand counted. This could potentially create less accurate data, and meant that less data was able to be produced. Regardless, the results were fairly consistent with what was expected.

Chapter 5

Conclusion

Both the model with further spreading fire and the model without show power-law behavior, but do not show scale invariance when measuring the mean number of clusters and mean cluster radius as functions of cluster size. Even when including the error and variance from the best fit lines, the slopes of the two power law functions are not close enough to support the presence of scale invariance. These results do not prove or disprove SOC in the model.

In the original definition of SOC [2], scale invariance was only needed between different system sizes for there to be Self-Organized Criticality, it was only in the Drossel-Schwabl model that claimed it would be robust across different parameter values. This data does not support scale invariance across parameter values, but in order to be more conclusive on whether or not there is SOC in the forest fire model, one would need to look at different system sizes within various set parameter values [8].

It would be helpful to have more data to support this. The big limitation here is that the data needed to be hand counted. By refining the cluster counting algorithm, it would become much easier to collect more data, and the results would be more accurate. It would also allow for a larger lattice, which would provide a wider range of results. In addition, the algorithm would eliminate human error in counting the

clusters.

In the future, it would be useful to adjust more parameters involved to support forest fires as a model of self-organized criticality. Some examples of this would be changing the lightning parameter and changing the forest density, then once again testing for the power-law behavior. Also, testing larger and smaller system sizes would be interesting because scale invariance is usually only referred to in that sense, while this paper tested other expanded parameters. This would show that these chaotic systems show the same trend over different sizes, which would make SOC a lot more applicable to real systems in nature. By expanding more parameters, the assumption is further supported.

In equilibrium systems, the overarching critical behavior only depends on large scale properties such as dimension and conservation laws, and not in the microscopic details [9]. This was further supported in this paper by changing a small scale factor, being the distance that the fire could spread from a single tree. These results aid to creating a future model that can assess the role of controlled burns that analyzes the importance of small scale forest properties [10]. Before that can happen, many more adjustments need to be made to the model such as thermodynamical properties and forest types.

Chapter 6

Appendix

Forest Fire Model

```
clear all

L=125; %Matriz dimensions
r=rand(L,L); %LxL matrix
p=.4; %Probability Density
z=r<p; %generates binary array
v=125;
s=.012; %lightning parameter probability
g=.2;

m1=1*z; %Take out of binary
r1= 0; %Start at all 0's
r2 = 0;

while r1 == 0 | r1 == L | r2 == 0 | r2 == L;
r1=randi(L); %Random number in Matrix
r2=randi(L);
```



```

        end

    for t=2:v
        m1(r1,r2)=2; %Random number becomes
        m=m1;
    end

    figure(1) %Figure at first time step
    [row, col] = find(m==1);
    position(:,1)=row;
    position(:,2)=col;
    plot(position(:,1),position(:,2),'og');

    hold on

    [row2, col2] = find(m==2);
    p2(:,1)=row2;
    p2(:,2)=col2;
    plot(p2(:,1),p2(:,2),'or');
    clear row col position row2 col2 p2
    hold off

%for n=1:4
    m = m1;

    for t=2:v %N Timesteps

```

```

q=zeros(L);

for i=2:(L-1)
for j=2:(L-1)
if m(i,j)==2;
    if m(i,j+1)==1
        m(i,j+1)=2;
    end
    if m(i,j-1)==1
        m(i,j-1)=2;
    end
    if m(i+1,j)==1
        m(i+1,j)=2;
    end
    if m(i-1,j)==1
        m(i-1,j)=2;
    end
    if m(i+1,j+1)==1
        m(i+1,j+1)=2;
    end
    if m(i+1,j-1)==1
        m(i+1,j-1)=2;
    end
    if m(i-1,j-1)==1
        m(i-1,j-1)=2;
    end
    if m(i-1,j+1)==1

```

```

        m(i-1,j+1)=2;
    end
end
end
end
% this section takes care of the edges
i=1;
for j=2:L-1
    if m(i,j)==2
        if m(i,j+1)==1
            m(i,j+1)=2;
        end
        if m(i,j-1)==1
            m(i,j-1)=2;
        end
        if m(i+1,j)==1 % NOTE - this is the left most edge, there are no
            m(i+1,j)=2;
        end
    end

    end
end
j=1;
for i=2:L-1
    if m(i,j)==2
        if m(i-1,j)==1
            m(i-1,j)=2;
        end
    end
end

```

```

        if m(i+1,j)==1
            m(i+1,j)=2;
        end
        if m(i,j+1)==1;
            m(i,j+1)=2;
        end
    end
end
end
i=L;
for j=2:L-1
    if m(i,j)==2
        if m(i,j+1)==1
            m(i,j+1)=2;
        end
        if m(i,j-1)==1
            m(i,j-1)=2;
        end
        if m(i-1,j)==1
            m(i-1,j)=2;
        end
    end
end
end
j=L;
for i=2:L-1
    if m(i,j)==2
        if m(i+1,j)==1
            m(i+1,j)=2;
        end
    end
end

```

```

        end
        if m(i-1,j)==1
            m(i-1,j)=2;
        end
        if m(i,j-1)==1
            m(i,j-1)=2;
        end
    end
end

end

%This section takes care of corners
i=1;
for j=1
    if m(i,j)==2
        if m(i+1,j)==1
            m(i+1,j)=2;
        end
        if m(i,j+1)==1
            m(i,j+1)=2;
        end
    end
end

end

j=1;
for i=L
    if m(i,j)==2
        if m(i-1,j)==1
            m(i-1,j)=2;
        end
    end
end

```

```

        if m(i,j+1)==1
            m(i,j+1)=2;
        end
    end
end
i=L;
for j=1
    if m(i,j)==2
        if m(i-1,j)==1
            m(i-1,j)=2;
        end
        if m(i,j+1)==1
            m(i,j+1)=2;
        end
    end
end
j=L;
for i=L
    if m(i,j)==2
        if m(i-1,j)==1
            m(i-1,j)=2;
        end
        if m(i,j-1)==1
            m(i,j-1)=2;
        end
    end
end
end

```

```

if (rand<g)
    m(randi(L),randi(L))=2;
end

% UNCOMMENT THESE LINES IF YOU WANT TO SEE THE TREES AT EACH TIME STEP
% IF YOU WANT TO SEE JUST THE LAST TIME STEP change figure(t) to figure(v)
hold on

    figure(t)
    [row, col] = find(m==1);
    position(:,1)=row;
    position(:,2)=col;
    plot(position(:,1),position(:,2),'og');

    hold on

    [row2, col2] = find(m==2);
    p2(:,1)=row2;
    p2(:,2)=col2;
    plot(p2(:,1),p2(:,2),'or');

    clear row col position row2 col2 p2

end

Cluster Counting Code

A = [1,2,1;1,1,2;2,2,2;2,2,1;1,1,1];

```

```
B = find(A==1);
cc=1;
for t=1:size(B,1)
    nearn = [B(t);B(t)+1;B(t)-1;B(t)+5;B(t)-5]; %5 is the number of rows in A
    C=intersect(B,nearn);
    if size(C,1) > 1
        cc=cc+1;
        clusters=C;
    end
end
end
```


References

- [1] B. Drossel and F. Schwabl, Phys. Rev. Lett. **69**, 1629 (1992).
- [2] P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, (1987).
- [3] R. Frigg, .
- [4] S. Clar, B. Drossel, and F. Schwabl, Physial Review E Phys **50**, 1009 (1994).
- [5] P. Bak and M. C. K. Chen, Nature 780 (1989).
- [6] S.Clar, B.Drossel, and F.Schwabl, Journal of Physics: Condensed Matter J. Phys **37**, 6803 (1996).
- [7] P. Grassberger, **17**, (2002).
- [8] B. Drossel, S. Clar, and F. Schwabl, The European Physical Journal B **15**, 177 (2000).
- [9] S. Clar, B. Drossel, and F. Schwabl, J. Phys **8**, 6803 (1996).
- [10] B. Malamud, G. Morein, and D. Turcotte, Science **281**, 1840 (1998).